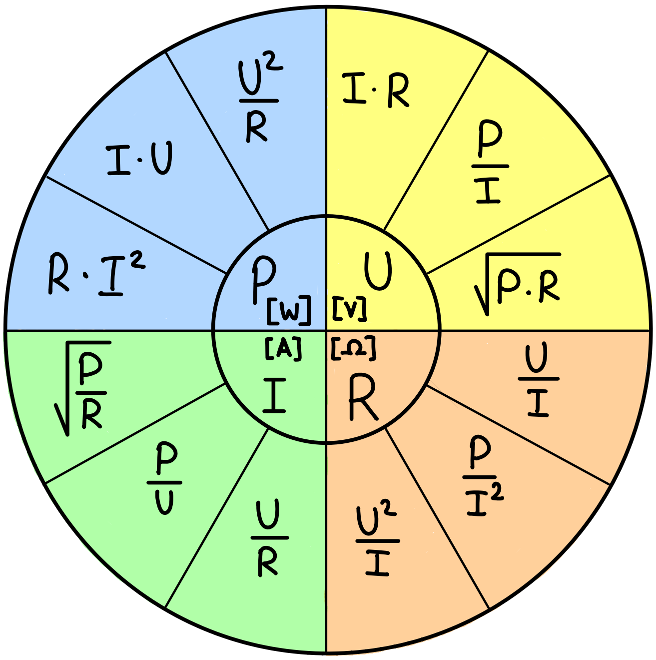
Introduction

Decimal prefixes

T	tera	10^{12}	μ	micro	10^{-6}
G	giga	10^{9}	n	nano	10^{-9}
M	mega	10^{6}	p	pico	10^{-12}

ET basis



Current strenght (I)

$$I = \frac{\text{electric charge}}{t} \; [A]$$

Current density (J)

$$J = \frac{I}{F} \; [A/mm^2] \; ; \; F = \text{cross section}$$

In houses, $J = 2 \dots 10 \; A/mm^2$

Resistance

$$R = \frac{U}{I} \; [\Omega]$$

Kirchhoff's current law (KLC)

$$\sum_{k=1}^{n} \underline{I_k} = \underline{I_1} + \underline{I_2} + \dots + \underline{I_n} = 0$$

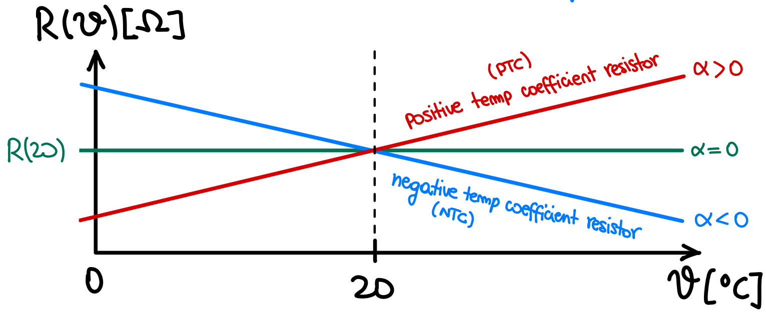
Kirchhoff's voltage law (KVC)

$$\sum_{k=1}^{n} \underline{U_k} = \underline{U_1} + \underline{U_2} + \dots + \underline{U_n} = 0$$

Voltage drop/divider formula

$$\boxed{U_0 = U_1 \frac{R_2}{R_1 + R_2} \; ; \; U_0 = \frac{R_2 U_1 + R_1 U_2}{R_1 + R_2}}$$

Resistance temperature dependance



$$\boxed{R(\vartheta) = R_{20}\left(1 + \alpha(\vartheta - 20^\circ)\right) = R_{20}(1 + \alpha \cdot \Delta T)}$$

$$\alpha = \left[\frac{1}{K}\right] \rightarrow \alpha_{copper} = 0{,}0039 \; 1/K$$

Material properties

Specific resistance (ρ)

$$\rho = R \cdot \frac{A}{l} \; \left[\frac{\Omega mm^2}{m}\right] = 10^{-6} \, \Omega m$$

$$\rho_{copper} = 0{,}01786 \; \frac{\Omega mm^2}{m}$$

Conductance (G)

G is the reciprocal of R:

$$G = \frac{I}{U} = \frac{1}{R} \; [S \text{ (Siemens)}]$$

Specific conductivity (γ)

$$\rho = \frac{1}{\gamma} \Rightarrow \gamma = G \cdot \frac{l}{A} \; \left[\frac{S \cdot m}{mm^2}\right] = 10^6 \, \frac{S}{m}$$

$$\gamma_{copper} = 56 \; \frac{S \cdot m}{mm^2}$$

Resistors in series connection

$$I = I_{TOT} = I_{R1} = I_{R2} = I_{Rn}$$

$$R_{eq} = \frac{U_{TOT}}{I} = \frac{U_{R1} + \dots + U_{Rn}}{I} = R_1 + \dots + R_n$$

Resistors in parallel connection

$$U = U_{R1} = U_{R2} = U_{Rn}$$

$$R_{eq} = \frac{U}{I_{TOT}} = \frac{U}{I_{R1} + \dots + I_{Rn}} = \frac{1}{\frac{1}{R_1} + \dots + \frac{1}{R_n}}$$

$$G_{eq} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \dots + \frac{1}{R_n}$$

Parallel connection of TWO resistors

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

Linear voltage sources & superposition

Resistance particular cases

- $R = 0\,\Omega \rightarrow$ no resistance, perfect wire
- $R = \infty\,\Omega \rightarrow$ no connection

Linear voltage source

Open circuit

$$R_a = \infty \Rightarrow I = 0\ ;\ U = U_q$$

Short circuit

$$R_a = 0 \Rightarrow I = I_K = \frac{U_q}{R_i}\ ;\ U = 0$$

$$\hookrightarrow R_i = \frac{U_q}{I_K} = \frac{\text{source voltage}}{\text{short circ. current}}$$

Normal operating conditions

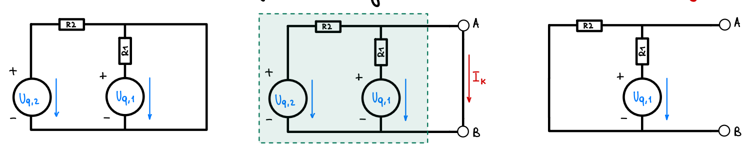
$$0 < R_a < \infty$$

$$\hookrightarrow U = U_q - U_i = U_q - I \cdot R_i = I \cdot R_a$$

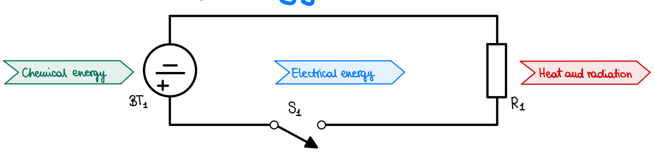
$$\hookrightarrow R_i = \frac{-\Delta U}{\Delta I} = \frac{-\text{change of terminal voltage}}{\text{change of load current}}$$

Superposition procedure

1. Select one source and switch off all others ($U = U_q$, $I = I_K$);
2. Calculate all voltages and currents cause by the chosen single source;
3. Repeat 1. and 2. for each source;
4. Add the contributions of all voltages (Attention to the direction & signs!).



Conservation of energy in an electrical circuit



Efficiency (η)

$$\boxed{\eta = \frac{P_{IN}}{P_{OUT}} \leq 1}$$

Conversion of power to mechanical power

$$P_{el} = U \cdot I \rightarrow \text{Motor} \rightarrow P_{mech} = U \cdot I \cdot \eta = M \cdot \omega$$

$$\downarrow$$

$$P_{loss} = U \cdot I \cdot (1 - \eta)$$

Conversion of electrical energy to heat energy

$$W_{el} = P_{el} \cdot t = \frac{m \cdot c \cdot \Delta\vartheta}{\eta}$$

Energy costs

$$\text{costs} = W \cdot k\ ,\ k = \text{energy tariff}\ \left(\sim 0{,}20\ \frac{CHF}{kWh}\right)$$

Fields, storage & capacitor + capacitance

Coulumb's law

It calculates the amount of force between two electrically charged particle

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 \cdot q_2}{r^2}\ ;\ \varepsilon_0 = 8{,}854 \cdot 10^{-12}\ \frac{As}{Vm}$$

Electric field and force on a charge Q

Homogeneous el. fields: $E = \dfrac{U}{d}$ — E = el. field strength [V/m], d = distance of the electrodes [m]

Force on a point charge: $F = Q \cdot E$ — F = force [N], Q = charge [As]

Capacitance of a plate capacitor

$$C = \varepsilon \cdot \frac{A}{d}\ \ [F = \text{Farad}]$$

Permittivity

$$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$$

ε_0 = absolute permittivity ($8{,}854 \cdot 10^{-12}$ As/Vm)

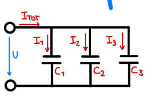
ε_r = relative permittivity of the dieletric, relative to the air

Energy in a capacitor

$$\int_0^{t_{empty}} U(t) \cdot I\, dt = \frac{1}{2} U_0 \cdot I \cdot t_{empty} = \frac{1}{2} C \cdot U_0^2$$

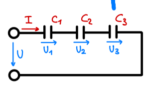
$$\hookrightarrow U(t) = U_0 \left(1 - \frac{t}{t_{empty}}\right)$$

Capacitors in parallel connection



$$C_{equi} = \frac{\sum_n Q_n}{U} = \frac{\varepsilon \cdot (\sum_n A_n)}{d} = \sum_n C_n = C_1 + C_2 + C_3$$

Capacitors in series connection



$$\frac{1}{C_{equi}} = \sum_n \frac{1}{C_n} = \frac{Q}{\sum_n U_n} = \frac{\varepsilon \cdot A}{\sum_n d_n}$$

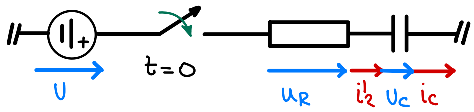
Ideal capacitor – voltage relationship

$$i = C \cdot \frac{\Delta u}{\Delta t}$$

! $\rightarrow$ Voltage CANNOT instantly jump from one value to another in a capacitor

CR series circuit

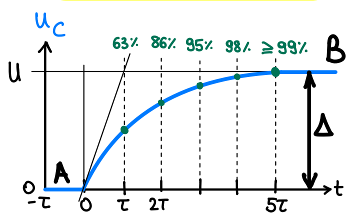
$$!\ \begin{cases} U = u_R + u_C \\ i_R = i_C \end{cases}$$

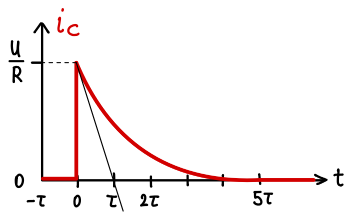


Transient analysis in RC circuits

CR time constant

$$\boxed{\tau = R \cdot C}$$





Capacitor charging

$$U_c(t) = U_0\left(1 - e^{-t/\tau}\right)$$

$$I_c(t) = \frac{U_0}{R} e^{-t/\tau}$$

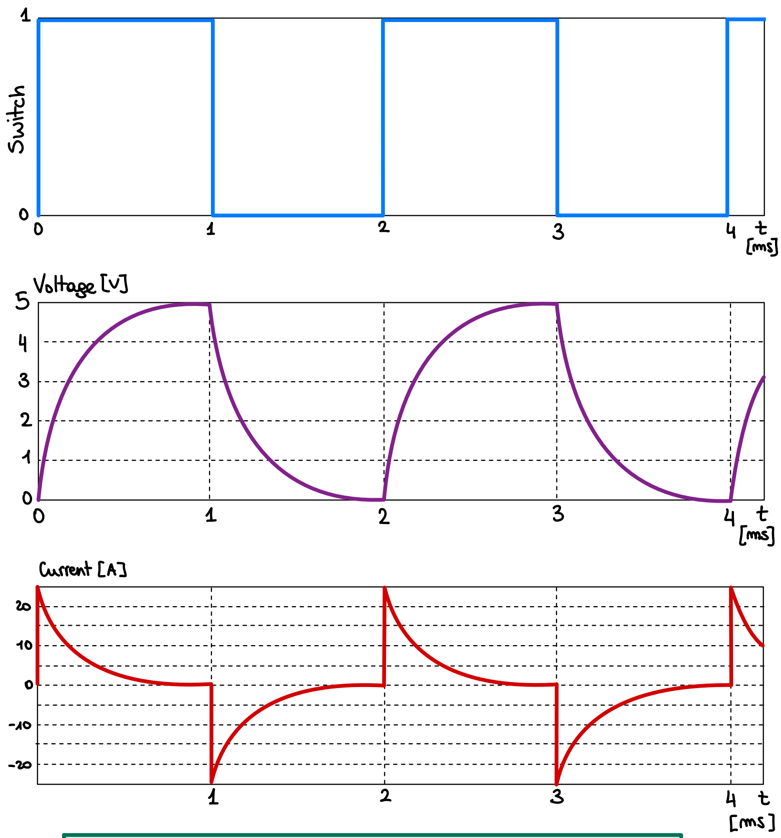
Capacitor discharging

$$U_c(t) = U_0 \cdot e^{-t/\tau}$$

$$I_c(t) = -\frac{U_0}{R} e^{-t/\tau}$$

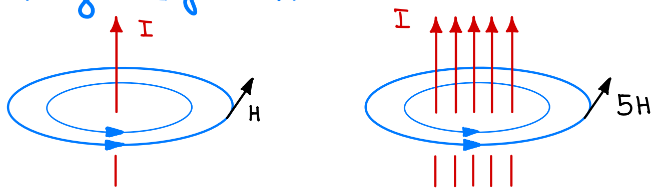
Transitional phase ($0\tau < t < 5\tau$)

$$\boxed{f(t) = A + \Delta\left(1 - e^{-t/\tau}\right) = A + (B - A)\left(1 - e^{-t/\tau}\right)}$$



Fields, storage & inductor + inductance

Magnetic field H



Magnetomotive force (current density)

$\Theta = N \cdot I$; N = number of turns

Ampère's circuital law

$$\Theta = \oint \overrightarrow{H(s)}\, d\vec{s} = N \cdot I = H \cdot \ell$$

Magnetic flux density

$$B = \frac{\phi}{A} = \mu \cdot H = \mu_0 \mu_r \cdot H$$

$$\left[\frac{Vs}{m^2} = T = \frac{Wb}{m^2} = \frac{H}{m} \cdot \frac{A}{m} = \frac{4\pi \cdot 10^{-7}\, Vs}{m} \cdot [-] \cdot \frac{A}{m}\right]$$

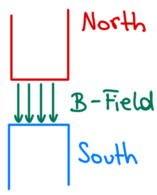
$\phi = \sum$ B-lines = magnetic flux | μ_0 = magnetic constant $= 4\pi \cdot 10^{-7}$

μ = magnetic permeability

H = magnetic field strength | μ_r = relative permeability

Magnetic flux ϕ

$\phi = B \cdot A$

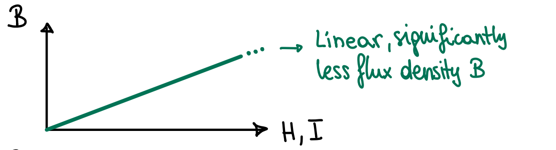


Different μ_r

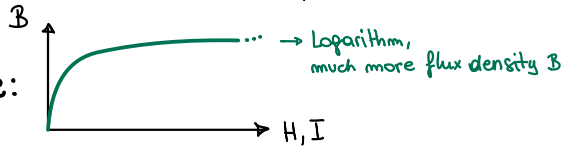
Material	μ_r
Air	1
Pure iron	≤250'000
Electrical steel	500...7000
Steel	40...7000
Water	0.99991

Coils with and without iron core

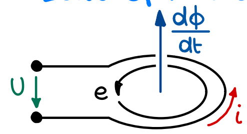
Air core:



Iron core:



Law of induction and inductance



$$U = -N \cdot \frac{d\phi}{dt}$$

Stationary case: $\frac{\Delta i}{\Delta t} = 0 \Rightarrow u = 0\,V$

Inductance and inductor

Inductance (L) is the capability to generate magnetic fields

$$L = \frac{N \cdot \phi}{I} = U \cdot \frac{\Delta t}{\Delta I} \qquad \left[H\ (\text{Henry}) = \frac{[-] \cdot Wb}{A} = \frac{Vs}{A}\right]$$

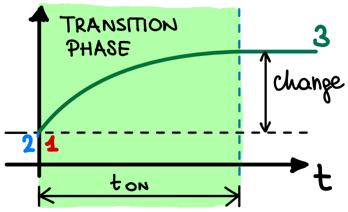
Energy in the magnetic field

$W = \frac{1}{2} L I^2$ → The current in a coil cannot change abruptly

Current-voltage relationship of an ideal coil

$u = L \cdot \Delta i / \Delta t$ → Special case: $0 = \Delta i/\Delta t \rightarrow u = 0$

Transient analysis in RL circuits



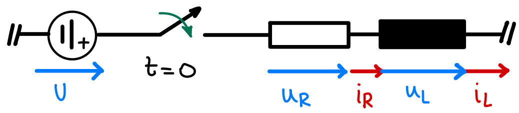
1: Starting value $y(0+)$
2: State before switching $y(0-)$
3: Final value $y(\infty)$

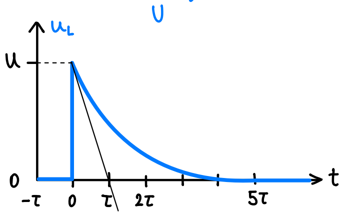
Constant τ: $\tau = L \cdot R_{TOT}^{-1}$

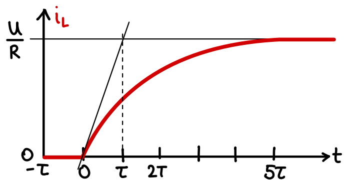
Steps:

1) $y(0-) = i_L(0-)$ 2) $y(0+) = i_L(0) = i_L(-0)$ since it cannot jump!
3) $y(\infty) = i_L(\infty) = i_L(5\tau)$
4) $y(t) = i_L(t) = i(\infty) + (i_L(0+) - i_L(\infty))e^{-t/\tau}$

Charging an inductor in a RL-network

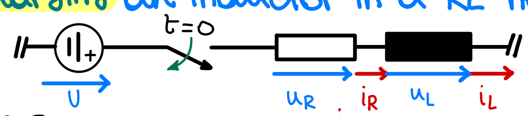


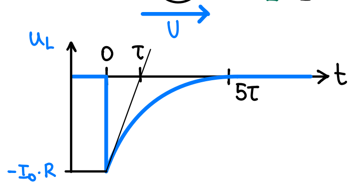


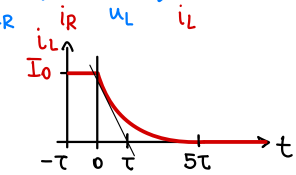


$$\boxed{u_L = U \cdot \exp\left(-\frac{t}{\tau}\right)} \qquad \boxed{i_L = \frac{U}{R}\left(1 - \exp\left(-\frac{t}{\tau}\right)\right)}$$

Discharging an inductor in a RL-network

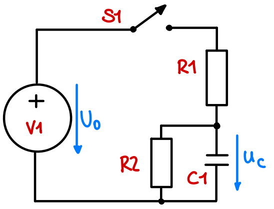






$$\boxed{u_L = -I_0 R \cdot \exp\left(-\frac{t}{\tau}\right)} \qquad \boxed{i_L = I_0 \exp\left(-\frac{t}{\tau}\right)}$$

Repetition



$u_c(0)=0$

$u_c(\infty)=U_0\dfrac{R_2}{R_1+R_2}=\Delta u_c$

$\tau = C\,(R_1 \| R_2)$

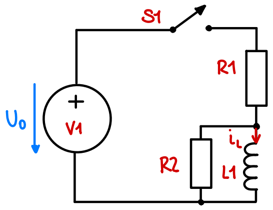
$u_c(t)=u_c(\infty)\left(1-\exp(-t/\tau)\right)$

$i_L(0)=0$

$i_L(\infty)=\dfrac{U_0}{R_1}=\Delta i_L$

$i_L(t)=i_L(\infty)\cdot\left(1-\exp(-t/\tau)\right)$

$\tau=\dfrac{L}{R_1 \| R_2}$



Sinusoidal, effective value, phasors

Voltage oscillation as a function of the angle

$$u(t)=\hat{U}\sin(\omega\cdot t)$$

Zero phase angle φ and phase shift

$u(t)=\hat{U}\sin(\omega t+\varphi_u)$

$i(t)=\hat{I}\sin(\omega t+\varphi_i)$

$\Delta\varphi=\varphi_u-\varphi_i$

Power in sin signal and effective value

$$p(t)=u(t)\cdot i(t)=\frac{u(t)^2}{R}=i(t)^2\cdot R$$

Active power

$$P(t)=\frac{1}{T}\int_0^T p(t)\,dt$$

Effective value

$$U_{eff}=\sqrt{\frac{1}{T}\int_0^T u(t)^2\,dt}=\frac{\hat{U}}{\sqrt{2}}=\hat{U}\cdot 2^{-1/2}$$

$$I_{eff}=\dots=\hat{I}/\sqrt{2}$$

Inductive and capacitive reactance

Relationship between current and voltage

on a capacitor: $i=C\cdot\dfrac{\Delta u}{\Delta t}$

on an ideal inductor: $u=L\cdot\dfrac{\Delta i}{\Delta t}$

!

Current waveform

through a capacitor:

Voltage is behind (lags) the current by 90°

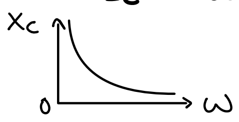
through an inductor:

Voltage leads the current by 90°

Reactance

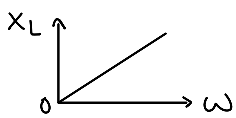
Capacitive

$X_C=\dfrac{U_C}{I_C}=\dfrac{1\angle-90°}{\omega\cdot C}$



Reactive

$X_L=\dfrac{U_L}{I_L}=\omega L\angle 90°$

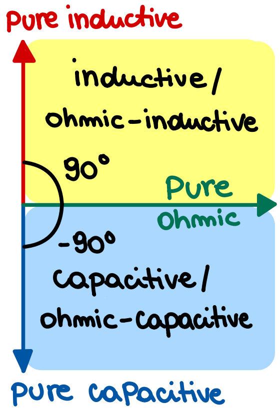


Impedance and admittance

Impedance

$$Z\angle(\varphi_U-\varphi_i)=\frac{u(t)\angle\varphi_u}{i(t)\angle\varphi_i}$$

Type of impedance



Circuite type	$Z=$	φ_Z	P	Q
purely resistive	R	0°	>0	0
purely inductive	$X_L\angle\varphi_L$	+90°	0	>0
purely capacitive	$X_C\angle\varphi_C$	-90°	0	<0
resistive - inductive	$R+X_L\angle\varphi_L$	$0°<\varphi_L<+90°$	>0	>0
resistive - capacitive	$R+X_C\angle\varphi_C$	$-90°<\varphi_C<0°$	>0	<0

Phase shifts

capacitance: $\varphi=\tan^{-1}\left(\dfrac{-X_C}{R}\right)$

reactivity: $\varphi=\tan^{-1}\left(\dfrac{X_L}{R}\right)$

Series connection

Resistances

$$R_{eq}=\frac{\Sigma_i U_{Ri}}{I}=\Sigma_i R_i$$

Impedances

$$Z_{eq}=\frac{\Sigma U_R+\Sigma U_C\varphi_C+\Sigma U_L\varphi_L}{I\angle 0°}=\Sigma_i Z_i$$

Parallel connection

Resistance

$$R_{eq}=\frac{U}{I}=\frac{U}{\Sigma_i I_{Ri}}=\frac{1}{\Sigma_i\frac{1}{R_i}}$$

Impedances

$$Z_{eq}=\frac{\underline{U}}{\underline{I}}=\frac{U}{I\angle 0°}=\frac{1}{\Sigma_i\frac{1}{Z_i}}$$

Voltage and current phasor

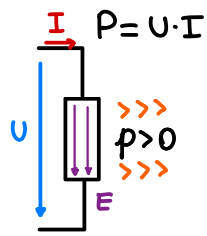
$$\underline{U}=\underline{Z_{element}}\cdot\underline{I}$$

Power in AC, power factor

Power in electrical circuits

Passive devices (loads)

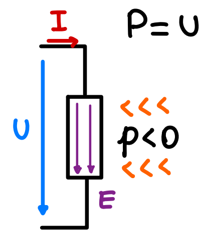
$P=U\cdot I>0$



Electrical energy
↓
other form (eg. heat)

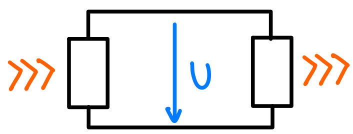
Actives devices (power sources)

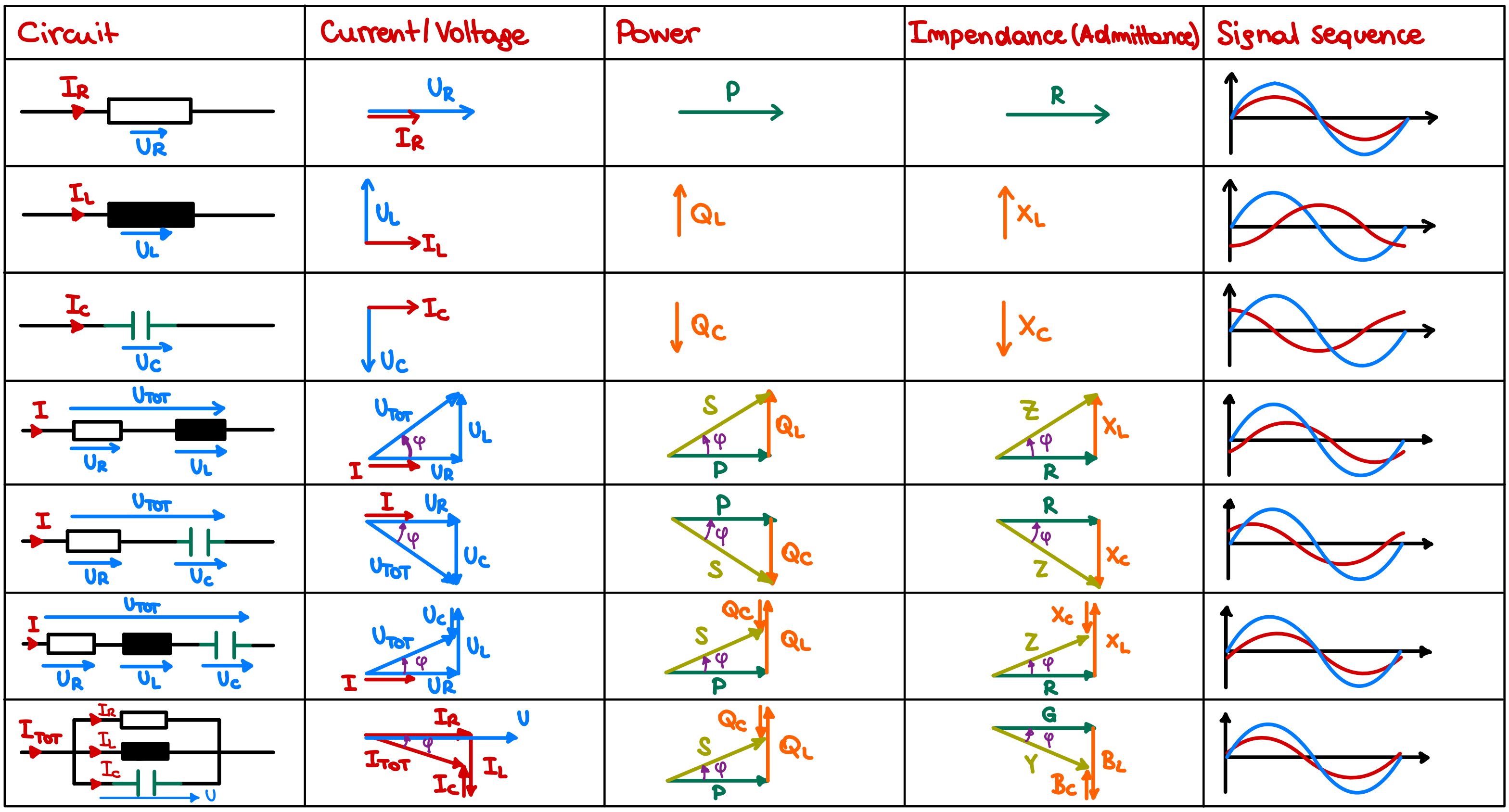
$P=U\cdot I<0$



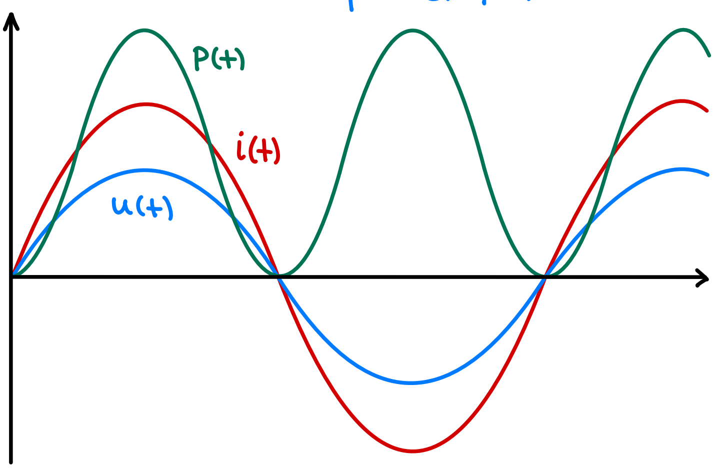
other energy
↓
Electrical energy + power network supply

Conservation of energy





Instantaneous power p(t)



Real power on R

if $\varphi = 0°$

$P = U \cdot I$

if $\varphi = \pm 90°$

$P = 0$

AC Powers

Apparent power [VA]

$S = U \cdot I$

$S = \sqrt{P^2 + Q^2}$

Average power [W]

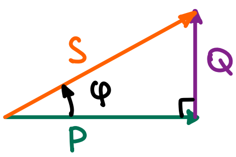
$P = U \cdot I \cdot \cos\varphi$

$P = S \cdot \cos\varphi$

Reactive power [var]

$Q = U \cdot I \cdot \sin\varphi$

$Q = S \cdot \sin\varphi$



Power factor

$\cos\varphi = \frac{P}{S}$

Performance factor

$\lambda = \left|\frac{P}{S}\right| = |\cos\varphi|$

	Pure Ohmic	Pure capacitive	Pure inductive	Arbitrary impedance
Impendance Z	$\underline{Z} = R\angle 0°$ (I_R, U_R)	$\underline{Z} = X_C\angle -90°$ (I_C, U_C)	$\underline{Z} = X_L\angle +90°$ (I_L, U_L)	$\underline{Z} = Z\angle\varphi_Z$ (I_Z, U_Z)
Real power P	$P_R = U_R \cdot I_R = \frac{U_R^2}{R} = I_R^2 \cdot R$	$P_C = 0\,W$	$P_L = 0\,W$	$P_Z = U_Z \cdot I_Z \cdot \cos(\varphi_Z)$
Reactive power Q	$Q_R = 0\,var$	Negative! $Q_C = -U_C \cdot I_C = -I_C^2 \cdot X_C = -\frac{U_C^2}{X_C}$	$Q_L = U_L \cdot I_L = I_L^2 \cdot X_L = \frac{U_L^2}{X_L}$	$Q_Z = U_Z \cdot I_Z \cdot \sin(\varphi_Z)$
Apparent power S	$S_R = P$	$S_C = \lvert Q_C\rvert$	$S_L = \lvert Q_L\rvert$	$S_Z = U_Z \cdot I_Z$

Work and Energy

Real energy

$W_W = P \cdot t$

Reactive en.

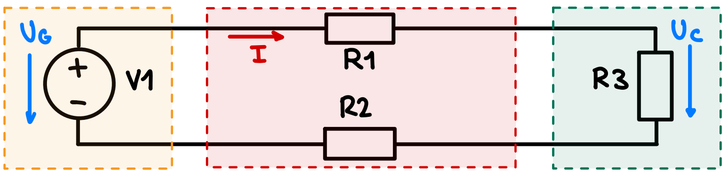
$W_B = Q \cdot t$

AC generator

$U = -N \cdot \frac{\Delta\Phi}{\Delta t}$

Three-phase circuits

Power transport

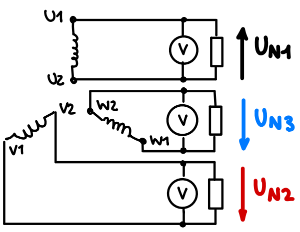


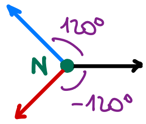
Producer	Transmission	Customer
$P_{PRO} = U_G \cdot I$	$P_{TRANS} = (R_1 + R_2) \cdot I^2$	$P_{CUS} = U_C \cdot I$

$$\eta = \frac{P_{CUS}}{P_{PRO}} = \frac{P_{PRO} - P_{TRANS}}{P_{PRO}} = 1 - \frac{P_{TRANS}}{P_{PRO}}$$

The loss in the line decreases with the square of the U_{TRANS}

Generator





$U_{N1} = U_S \angle 0°$

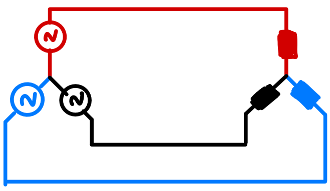
$U_{N2} = U_S \angle -120°$

$U_{N3} = U_S \angle 120°$

$\underline{U_{N1}} + \underline{U_{N2}} + \underline{U_{N3}} = 0$

Wye connection

3 connections instead of 6



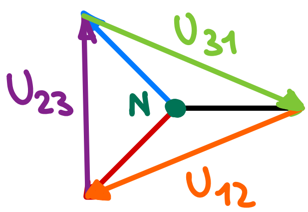
Line voltage → The 3ph sys provides 2 voltages

Phase voltage

$U_{N1} = U_S \cdot \sin(\omega t + 0°)$

$U_{N2} = U_S \cdot \sin(\omega t - 120°)$

$U_{N3} = U_S \cdot \sin(\omega t + 120°)$



Line voltage

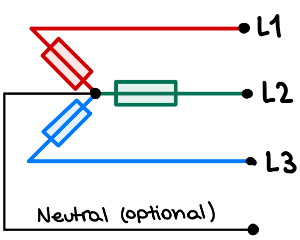
$U_{12} = U_{N2} - U_{N1} = \sqrt{3}\, U_S \cdot \sin(\omega t - 150°)$

$U_{23} = U_{N3} - U_{N2} = \sqrt{3}\, U_S \cdot \sin(\omega t + 90°)$

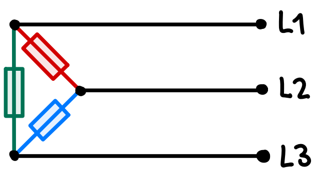
$U_{31} = U_{N1} - U_{N3} = \sqrt{3}\, U_S \cdot \sin(\omega t - 30°)$

Wye (Y) and delta (Δ) circuits

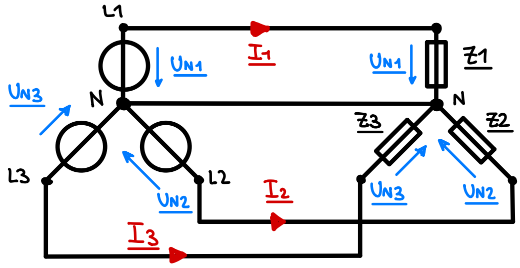
Y-config



Δ-config



Y-circuit



- $Z1 = Z2 = Z3$
- $I = I_S$
- $|I_1| = |I_2| = |I_3|$
- $U = \sqrt{3}\, U_S$

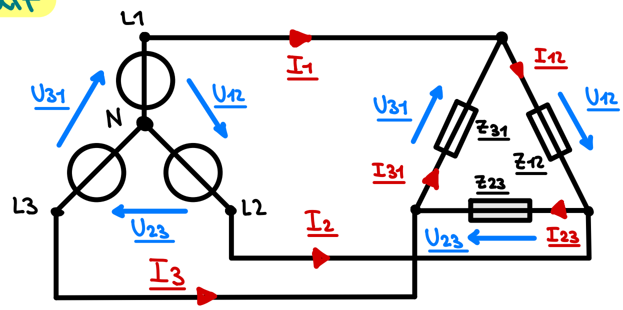
$S_1 = U_{N1} \cdot I_1$

$S_2 = U_{N2} \cdot I_2$

$S_3 = U_{N3} \cdot I_3$

$S_{TOT} = 3 \cdot U_S \cdot I = \sqrt{3}\, U I$

Δ-circuit



- $\underline{Z_{12}} = \underline{Z_{23}} = \underline{Z_{31}}$
- $I = \sqrt{3}\, I_S$
- $|I_{12}| = |I_{23}| = |I_{31}|$
  - $= \frac{I_1}{\sqrt{3}} = \frac{I_2}{\sqrt{3}} = \frac{I_3}{\sqrt{3}}$
- $U = \sqrt{3}\, U_S$

$S_{12} = U_{12} \cdot I_{12} = \sqrt{3}\, U_{N1} \cdot \frac{I_1}{\sqrt{3}}$

$S_{23} = U_{23} \cdot I_{23} = \sqrt{3}\, U_{N2} \cdot \frac{I_2}{\sqrt{3}}$

$S_{31} = U_{31} \cdot I_{31} = \sqrt{3}\, U_{N3} \cdot \frac{I_3}{\sqrt{3}}$

$S_{TOT} = 3 \cdot U_S \cdot I = \sqrt{3} \cdot U \cdot I$

Power calculation

Apparent power [VA]

$S = 3 \cdot U_S \cdot I = \sqrt{3} \cdot U \cdot I$

Average power [W]

$P = S \cdot \cos\varphi$

Reactive power [var]

$Q = S \cdot \sin\varphi$

Average power of resistive loads [W]

$$\left.\begin{aligned} P_Y &= 3 \cdot \frac{U_S^2}{R} = \frac{(\sqrt{3}\, U_S)^2}{R} = \frac{U^2}{R} \\ P_\Delta &= 3 \cdot \frac{(\sqrt{3}\, U_S)^2}{R} = 3\frac{U^2}{R} \end{aligned}\right\} P_Y = P_\Delta$$