Mathematics 1A Tutoring session
Semester Week 5

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Exercises

1 Trigonometry

Exercise 1.1:
Find the hypotenuse of a right triangle with adjacend side $b = 36.4$ cm and opposite side $c = 41.8$ cm.

Exercise 1.2:
Solve a right triangle where only the hypothenuse $a = 3$ cm and the angle $\beta = 34,7°$ are known.

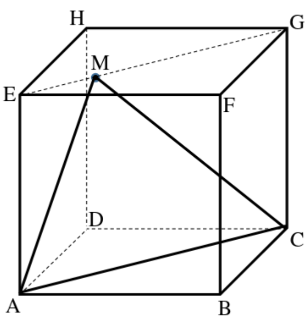
Exercise 1.3:
Determine domain and range of the following trigonometric functions:

$$\sin(x) \quad ; \quad \cos^{-1}(x) \quad ; \quad \tan(x) \quad ; \quad \arctan(x)$$

Exercise 1.4:
A block of mass $m = 2\,\text{kg}$ rests on an inclined plane that forms an angle θ with the horizontal. The gravitational acceleration is $g = 9.81\,\text{m}\,\text{s}^{-2}$.

a. Express the components of the force parallel and perpendicular to the plane, $F_\parallel$ and $F_\perp$, as functions of θ, knowing that $F = mg$.
b. Compute $F_\parallel$ and $F_\perp$ for $\theta = 25°$.
c. Find the value of θ for which $F_\parallel = \frac{1}{2}F_\perp$.
d. Determine the length L of the inclined plane knowing that the height is $h = 1.5\,\text{m}$ and the horizontal projection is $x = 3.3\,\text{m}$.

Exercise 1.5 BONUS:



A cube $ABCDEFGH$ with edge length 8 cm is given.
Point M is defined so that $\overrightarrow{EM} = \frac{2}{5}\,\overrightarrow{EG}$.
Calculate the measure of the sides and angles of triangle ACM

2 Exponential functions

Exercise 2.1:
Determine the half-life of a radioactive substance X, knowing that after one year its mass has decreased to one third.

Exercise 2.2:
A sample of St-89 (Strontium-89, a radioactive element with $T = 50.5$ days) has lost 2 g of mass after one month. Determine the mass of the same initial sample after one year.

Hint: for decays, $k = -\dfrac{\ln(2)}{T}$

Exercise 2.3:
Solve in $\mathbb{R}$ **without the calculator**:

$$\log\left(x^2\right) = 3 \quad ; \quad \ln(x+1) = \frac{1}{2} \quad ; \quad \ln(x+1) = 0.001^2 \quad ; \quad \log_2\left(x^2 - 4x + 4\right) = 2 \quad ; \quad \left(x^2 + x - 2\right)\ln(2x) = 0$$

Exercise 2.4:
A hot object with an initial temperature T_0, placed at time $t = 0$ in an environment with lower temperature T_1, cools according to Newton's law of cooling:

$$\boxed{T = T_1 + (T_0 - T_1)\, e^{-kt}}$$

where T is the temperature of the object at time t, and k is a constant depending on the material of the object.

Consider a steel sphere heated to 133°C and then placed to cool in a room where the air temperature is 22°C. Calculate:

a. the temperature of the sphere after 20 minutes, knowing that after 10 minutes it had cooled to 108°C;

b. the time required for the temperature to drop to 66.5°C.

3 Composite functions

Exercise 3.1:
Let $f : \mathbb{R} \to \mathbb{R}$

$$x \mapsto y = \begin{cases} \frac{1}{2}x + 1\ , & \text{for } x \geq 4 \\ x - 1\ , & \text{for } -1 < x < 4 \\ -2x - 4\ , & \text{for } x \leq -1 \end{cases}$$

Solve:

a. $f(0)$

b. $f(-11)$

c. $f\left(\frac{9}{2}\right)$

d. $f\left(f\left(-\frac{1}{2}\right)\right)$

c. $f(6) - f(-6)$

d. $f(3f(-1) - 2f(2))$

Exercise 3.2:
Let $f : \mathbb{R} \to \mathbb{R}$

$$x \mapsto y = \begin{cases} \frac{1}{2}x\ , & \text{for } x < -2 \\ \frac{2}{3}x + 1\ , & \text{for } -2 \leq x \leq 2 \\ \dfrac{x+2}{3}\ , & \text{for } x > 2 \end{cases}$$

Solve:

$$a.\ f\left(\frac{6}{5}\right) \quad ; \quad b.\ f\big(f(-3)\big) \quad ; \quad c.\ 2\big(f(7) - 1\big)^2$$

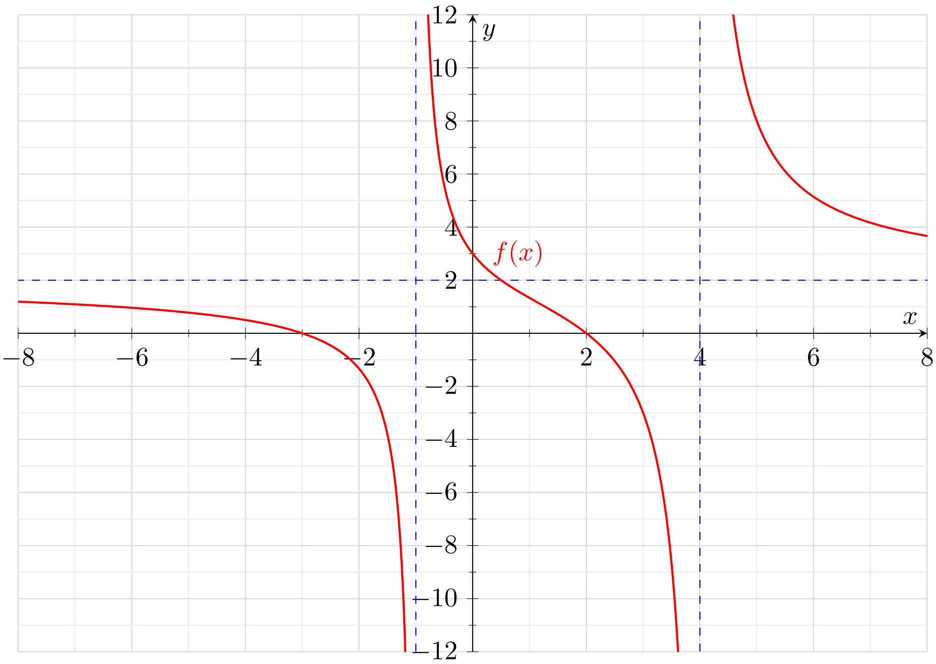
4 Limits

Exercise 4.1:
Solve the following limits using the dominant term method:

$$\lim_{x\to\infty} \frac{(x+1)^2}{x^2+1} \quad ; \quad \lim_{x\to\infty} \frac{\sin(x)}{x} \quad ; \quad \lim_{x\to\infty} \frac{1000x}{x^2+1} \quad ; \quad \lim_{x\to\infty} \frac{x^2-5x+1}{3x+7}$$

$$\lim_{x\to\infty} \frac{2x^2-x+3}{x^3-8x+5} \quad ; \quad \lim_{x\to\infty} \frac{(2x+3)^3\,(3x-2)^2}{x^5+5} \quad ; \quad \lim_{x\to\infty} \frac{2x^2-3x-4}{\sqrt[3]{x^4+1}}$$

Exercise 4.2:
For each of the following, determine whether the statement is true or false:



a) $\lim_{x\to\infty} f(x) = 0$, b) $\lim_{x\to 0} f(x) = 2$, c) $\lim_{x\to 4^-} f(x) = +\infty$, d) $\lim_{x\to -1} f(x) = f(-1)$

e) $\lim_{x\to 0} f(x) = 3$, f) $\lim_{x\to\infty} f(x) = -1$, g) $f(x) = 0 \iff x \in \{-3, 4\}$

Exercise 4.3 (From Question 4.1, Homeworks Week 4):
Are the following claims true or false? Explain why:

a. If a function y of x is not defined at the position $x = x_0$, then $\lim_{x\to x_0}$ does not exist either

b. If $\lim_{x\to x_0}$ does not exist, then y is also not defined at the point $x = x_0$

c. If a function y of x is defined at the position $x = x_0$, then $\lim_{x\to x_0} y = y\big|_{x=x_0}$

d. If x approaches 1.000.000 from the left, $1/x$ gets closer to the value 0. Therefore, $\lim_{x\to 1.000.000^-} \frac{1}{x} = 0$

e. If y has limit value 5 when x approaches 3 from the left, then y must already assume the value 5 in the range $x < 3$

Quick solutions

Solution 1.1:
a = 55.43 cm

Solution 1.2:
b = 1.71 cm, c = 2.47 cm, $\alpha = 55.3°$

Solution 1.3:

1. $\mathcal{D}_f = \forall x \in \mathbb{R}, \quad Im_f = [-1, 1]$
2. $\mathcal{D}_f = [-1, 1], \quad Im_f = [\pi, 0]$
3. $\mathcal{D}_f = \forall x \in \mathbb{R} \ \left\{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\right\}, \quad Im_f = [-1, 1]$
4. $\mathcal{D}_f = \forall x \in \mathbb{R}, \quad Im_f = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Solution 1.4:

a. $F_{\|} = mg\cos(x)\ , \quad F_{\perp} = mg\sin(x)$

b. $F_{\|} = 17.78$ N, $\quad F_{\perp} = 8.29$ N

c. $\theta \approx 1.11$ rad $= 63.4°$

d. $L = 3.6$ m

Solution 1.5:
$\overrightarrow{AC} = 8\sqrt{(2)}$ cm ≈ 11.3 cm

$\overrightarrow{AM} = \frac{8\sqrt{(33)}}{5}$ cm ≈ 9.19 cm

$\overrightarrow{MC} = \frac{8\sqrt{(43)}}{5}$ cm ≈ 10.49 cm

$\widehat{MAC} : \alpha \approx 60.5°$

$\widehat{ACM} : \beta \approx 49.7°$

$\widehat{AMC} : \gamma \approx 69.8°$

Solution 2.1:
Half-time: 0.63 years $\approx$ 230 days

Solution 2.2:
$m_0 = 5.93$ g

Solution 2.3:

a. $x = \pm\sqrt{10^3}$

b. $x = e^{\frac{1}{2}} - 1$

c. $x = e^{10^{-6}} - 1 \approx 0$

d. $x_1 = 0,\ x_2 = 4$

e. $x \in \left\{-2, \frac{1}{2}, 1\right\}$

Solution 2.4:

a. $T = 88.63°\text{C}$

b. $t \approx 35.2$ min

Solution 3.1:

a. $f(0) = 1$

b. $f(-11) = 18$

c. $f\left(\frac{9}{2}\right) = \frac{13}{4}$

d. $f\left(f\left(-\frac{1}{2}\right)\right) = -1$

e. $f(6) - f(-6) = -4$

f. $f\big(3f(-1) - 2f(2)\big) = 12$

Solution 3.2:

a. $f\left(\frac{6}{5}\right) = \frac{9}{5}$

b. $f\big(f(-3)\big) = 0$

c. $2\big(f(7) - 1\big)^2 = 8$

Solution 4.1:

a. $= 1$

b. $= 0$

c. $= 0$

d. $= 0$

e. $= 72$

f. $= \infty$

Solution 4.2:

a. False

b. False

c. False

d. False

e. True

f. False

g. False

Solution 4.3:
All the statements are False