1 Distance-dependence of IR temperature detection

1.1 Data of the worst-case experiment

The following worst-case temperature data are used:

x [cm]	T [°C]
5	50.00
10	45.00
20	41.50
30	36.50
40	29.75
50	27.00
60	26.00
70	24.75
80	24.25
90	24.00
100	24.00

The ambient temperature is $T_{\text{amb}} = 23.5°\text{C}$.

1.2 Regression exponential function

The distance dependence is now modeled as an exponential function of the form

$$T(x) = T_{\text{amb}} + Ae^{Bx} \Longleftrightarrow T(x) - T_{\text{amb}} = Ae^{Bx}.$$

Linearizing the model gives

$$\ln\big(T(x) - T_{\text{amb}}\big) = \ln A + Bx.$$

Defining

$$Y_i = \ln\big(T_i - T_{\text{amb}}\big),$$

the regression becomes linear:

$$Y_i = \alpha + Bx_i, \qquad \text{with} \qquad \alpha = \ln A.$$

The slope B is then given by

$$B = \frac{\sum_i x_i Y_i - \frac{1}{n}\left(\sum_i x_i\right)\left(\sum_i Y_i\right)}{\sum_i x_i^2 - \frac{1}{n}\left(\sum_i x_i\right)^2},$$

and the intercept α by

$$\alpha = \bar{Y} - B\bar{x},$$

where

$$\bar{x} = \frac{\sum_i x_i}{n}, \qquad \bar{Y} = \frac{\sum_i Y_i}{n}.$$

Finally,

$$A = e^{\alpha}.$$

Using the data in section 1.1, the fitted parameters are

$$A = 38.72, \qquad B = -0.04661,$$

which gives

$$T(x) = 23.5 + 38.72e^{-0.04661x}.$$

1.3 Final step function

To impose a conservative lower-bound behavior, the final model is defined piecewise as

$$T(x) = \begin{cases} 30, & \text{for } x \leq 38.28694, \\ 23.5 + 38.72e^{-0.04661x}, & \text{for } 38.28694 < x < 78.44575, \\ 24.5, & \text{for } x \geq 78.44575. \end{cases}$$

The first transition point was chosen such that

$$23.5 + 38.72e^{-0.04661x} = 30$$

at

$$x = 38.28694\,\text{cm},$$

and the second transition point such that

$$23.5 + 38.72e^{-0.04661x} = 24.5$$

at

$$x = 78.44575\,\text{cm}.$$

Thus, the piecewise function is continuous at both transition points.

1.4 Graphic