I. Heat transfer

Symbology

Symbol	Quantity	Unit
λ , k	Thermal conductivity	W/(m · K)
$\dot{q}$	Heat flux	W/m^2
$\dot{Q}$	Heat flow	W
α	Heat transfer coefficient	W/(m^2 · K)
a	Thermal diffusivity	m^2/s
ν	Kinematic viscosity	m^2/s

1. Types of heat transfer

1.1 Heat conduction

Occurs with all materials when there is a temperature gradient.

Steady-state conduction (1st Fourier's law):

$$\dot{q} = -\lambda \frac{\mathrm{d}T}{\mathrm{d}x}$$

Transient conduction (2nd Fourier's law 1D):

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial x^2}$$

1.2 Convective heat transfer

This occurs in moved fluids.

Newton's approach:

$$\dot{q} = \alpha (T_1 - T_2) = \frac{\lambda_{\text{fluid}}}{\delta_{\text{th}}} (T_1 - T_2)$$

1.3 Radiation

Bodies with a temperature above 0 K emit thermal radiation.

2. Heat conduction and conductivity

2.1 Heat flux

$$\dot{q} = -\lambda \frac{\mathrm{d}T}{\mathrm{d}x} = k\Delta T = \frac{\lambda}{\delta}\Delta T$$

Procedure:

$$\vec{\dot{q}} \int_0^\delta dx = -\lambda \int_{T_1}^{T_2} dT \Longrightarrow \dot{q} = \frac{\lambda}{\delta}(T_1 - T_2)$$

2.2 Heat flow

$$\dot{Q} = \frac{\Delta Q}{\Delta t} = \dot{q}A = kA\Delta T = \frac{\lambda}{\delta} A \Delta T$$

2.3 Thermal conduction k^{-1} and Thermal conductivity k

$$k_{\text{tot}}^{-1} = \sum_{i=1}^{n} \frac{\delta_i}{\lambda_i} \implies k_{\text{tot}} = \frac{1}{k_{\text{tot}}^{-1}}$$

3. Heat conductivity through a hollow cylinder

$$\dot{Q} = -\lambda A \frac{\mathrm{d}T}{\mathrm{d}r} = -\lambda 2\pi r l \frac{\mathrm{d}T}{\mathrm{d}r}$$

$$\boxed{\dot{Q} = 2\pi l \cdot \frac{T_0 - T_1}{\frac{1}{\lambda} \ln \frac{r_1}{r_0}} = 2\pi r_0 l k \Delta T}$$

With multilayered walls:

$$\dot{Q} = 2\pi l \cdot \frac{T_0 - T_n}{\sum_{i=1}^{n} \frac{1}{\lambda_i} \ln \frac{r_i}{r_{i-1}}} = \boxed{2\pi r_0 l k_0 (T_0 - T_n)}$$

3.1 Thermal conductivity k

$$k_0 = \frac{1}{r_0 \left(\left(\frac{1}{\lambda_1} \ln \frac{r_1}{r_0}\right) + \left(\frac{1}{\lambda_2} \ln \frac{r_2}{r_1}\right) + \cdots + \left(\frac{1}{\lambda_n} \ln \frac{r_n}{r_{n-1}}\right)\right)}$$

$$k_0^{-1} = r_0 \sum_n \left(\frac{1}{\lambda_n} \ln \frac{r_n}{r_{n-1}} \right)$$

3.2 Thin-walled pipe approximated as a flat wall

$$\dot{Q}_T = \lambda A \frac{T_0 - T_1}{\delta} \quad ; \quad A = 2\pi l \left(r_0 + \frac{\delta}{2}\right)$$

$$\frac{\dot{Q}_T}{\dot{Q}_W} = \frac{\delta}{\left(r_0 + \frac{\delta}{2}\right) \ln\left(1 + \frac{\delta}{r_0}\right)} = \frac{\delta}{r_0} \frac{1}{\left(1 + \frac{\delta}{2r_0}\right) \ln\left(1 + \frac{\delta}{r_0}\right)}$$

4. Convective heat transfer

4.1 Reynold's number

It compares inertial to viscous forces.

$$\boxed{Re = \rho c \frac{L_{\text{char}}}{\eta} = c \frac{L_{\text{char}}}{\nu}}$$

$Re < 2300$: Laminar flow;
$Re \approx 2300$: Transition zone;
$Re > 2300$: Turbulent flow

4.2 Prandtl number

It compares momemtum diffusivity to thermal diffusivity. Sets velocity's relative thickness and thermal boundary layers.

$$Pr = \frac{\nu}{a} = \frac{\delta_{\text{fluid}}}{\delta_{\text{th}}} = \frac{\eta c_p}{\lambda} \quad ; \quad a = \frac{\lambda}{\rho c_p} \quad ; \quad \nu = \frac{\eta}{\rho}$$

4.3 Nusselt number

Measures convective heat transfer relative to pure conduction across a boundary layer and the heat transfer coefficient α.

$$Nu = \frac{\alpha \cdot L_{\text{char}}}{\lambda_{\text{fl}}} = \dot{q} \frac{L_{\text{char}}}{\lambda \Delta T_{\text{char}}}$$

$$\alpha = \frac{Nu \cdot \lambda}{L_{\text{char}}}$$

4.3.1 Forced convection

$$\boxed{Nu = c \cdot Re^m \cdot Pr^n}$$

where c is a constant.

For laminar flow over a flat plate:

$$Nu = 0.664 \cdot \sqrt{Re} \cdot \sqrt[3]{Pr}$$

For a flow in the transition zone (Tz):

$$Nu_{\text{Tz}} = \sqrt{Nu_{\text{lam}}^2 + Nu_{\text{turb}}^2}$$

4.4 Grashof number

It compares buoyancy to viscous forces.

$$Gr = g\beta\Delta T \frac{L^3}{\nu^2} \quad ; \quad g = 9.81 \text{ N/kg}$$

$$\beta \Delta T = \frac{\rho_\infty - \rho_0}{\rho_\infty} = \frac{\Delta \rho}{\rho_\infty}$$

$$Gr = g \frac{\rho_\infty - \rho_0}{\rho_\infty} \frac{L^3}{\nu^2}$$

4.4.1 Natural convection

Peclet number

It compares advective to diffusive heat transport, i.e., the strength of convection relative to thermal diffusion.

$$Pe = RePr$$

Rayleigh number

It combines buoyancy driving and thermal diffusion, and it measures the intensity of natural convection.

$$Ra = GrPr$$

5. Heat exchanger

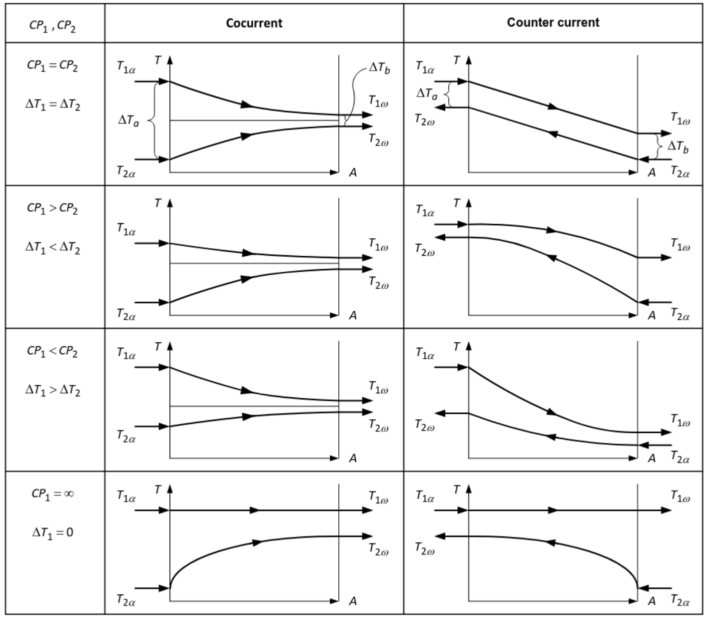
5.1 Energy balance

$$\dot{Q} = \dot{m} c_p \Delta\vartheta = CP\Delta\vartheta$$

$$\dot{Q}_1 = \dot{Q}_2$$

$$\dot{m}_1 c_{p1} \Delta\vartheta_1 = \dot{m}_2 c_{p2} \Delta\vartheta_2 \iff CP_1 \Delta\vartheta_1 = CP_2 \Delta\vartheta_2$$

5.2 Average temperature difference



5.2.1 Logarithmic mean temperature difference (LMTD)

$$\Delta T_m = \frac{\Delta T_a - \Delta T_b}{\ln \dfrac{\Delta T_a}{\Delta T_b}}$$

where:

$$\Delta T_a = |\vartheta_{1,a} - \vartheta_{2,a}| \quad ; \quad \Delta T_b = |\vartheta_{1,b} - \vartheta_{2,b}|$$

5.2.2 Mean temperature heat flow

$$\dot{Q} = kA\Delta T_m$$

5.2.3 Heat capacity rate ratio (Capacity ration)

It measures the relative ability of stream 1 vs. stream 2 to carry heat per kelvin:

$$R_1 = \frac{\Delta\vartheta_1}{\Delta\vartheta_2} = \frac{CP_1}{CP_2}$$

5.3 Cocurrent vs countercurrent flow heat exchanger

$$\dot{Q} = kA_{\text{co}}\Delta T_{m,\text{co}} = kA_{\text{count}}\Delta T_{m,\text{count}}$$

$$\frac{A_{\text{co}}}{A_{\text{count}}} = \frac{\Delta T_{m,\text{count}}}{\Delta T_{m,\text{co}}}$$

5.3.1 If $\dot{Q}_1 \neq \dot{Q}_2$

$$\frac{A_{\text{co}}}{A_{\text{count}}} = \frac{\dot{Q}_{\text{count}}\Delta T_{m,\text{count}}}{\dot{Q}_{\text{co}}\Delta T_{m,\text{co}}}$$

II. The 2nd law of thermodynamics

6. Recap of the 1st law

6.1 Closed systems

$$q_{12} + w_{12} = u_2 - u_1$$

6.2 Open systems

$$q_{12} + w_{12} = h_2 - h_1 + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)$$

$$w_{t12} = \int v\,dp + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right) + j_{12}$$

$$q_{12} + \int v\,dp + j_{12} = h_2 - h_1 \overset{\text{ideal gas}}{=\!=} c_p\left(T_2 - T_1\right)$$

7. The 2nd law

7.1 Statements

Energy = Exergy + Anergy

Processes	Reversible	Irreversible
Energy	Conserved	Reduced → Loss
Entropy	Constant	Increases → Production

7.2 Entropy

Entropy S: extensive state variable [J/K]

Specific entropy s: intensive state variable [J/kgK]

Entropy flow $\dot{S}$: entropy over time [W/K]

8. Entropy balance equation

$$dS = dS_Q + dS_m + dS_{irr}$$

8.1 Entropy balance terms

- Heat transfer across the system boundary:

$$dS_Q = \frac{dQ}{T}$$

- Mass flow across the system boundary:

$$dS_m = \dot{S} = \dot{m}\,s$$

- Irreversible processes inside the system:

$$\dot{S}_{irr}(t)\begin{cases} dS_{irr} > 0 & \text{irreversible processes} \\ dS_{irr} = 0 & \text{reversible processes} \\ dS_{irr} < 0 & \text{impossible} \end{cases}$$

8.2 Entropy balance equation for closed systems

$$dS = dS_Q + \cancelto{0}{dS_m} + dS_{irr}$$

$$dS = \dot{S}_Q\,dt + \dot{S}_{irr}\,dt = \left(\frac{\dot{Q}}{T} + \dot{S}_{irr}\right)dt$$

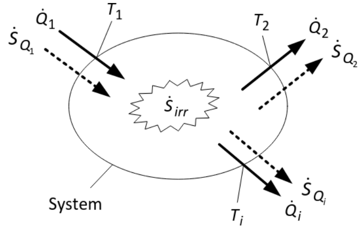
8.2.1 Closed system entropy flow balance equation

$$\frac{dS}{dt} = \sum_i \dot{S}_{Qi} + \dot{S}_{irr} \quad ; \quad \dot{S}_{irr} \geq 0$$

For steady-state case:

$$\dot{S}_{irr} = -\sum_i \dot{S}_{Qi} = -\sum_i \frac{\dot{Q}_i}{T_i} \geq 0$$

8.2.2 Visual representation



$$\sum_i \dot{S}_{Qi} = \sum_i \frac{\dot{Q}_i}{T_i} = -\dot{S}_{irr}$$

8.3 Entropy balance equation for open systems

$$\frac{dS}{dt} = \sum_\alpha \dot{m}_\alpha s_\alpha - \sum_\omega \dot{m}_\omega s_\omega + \sum \dot{S}_Q(t) + \dot{S}_{irr}(t)$$

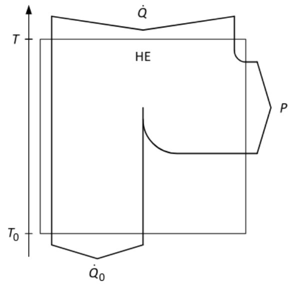
8.3.1 Steady-state flow processes

$$\dot{S}_{irr} = \sum_\omega \dot{m}_\omega s_\omega - \sum_\alpha \dot{m}_\alpha s_\alpha - \sum \dot{S}_Q \geq 0$$

8.3.2 Steady-state and adiabatic flow processes

$$\dot{S}_{irr} = \sum_\omega \dot{m}_\omega s_\omega - \sum_\alpha \dot{m}_\alpha s_\alpha \geq 0$$

9. The thermal enigne



$$\dot{Q} + \dot{Q}_0 + P = 0$$

$$\eta_{th} = \frac{-P}{\dot{Q}} = 1 - \frac{\left|\dot{Q}_0\right|}{\dot{Q}}$$

$$\dot{S}_{irr} = -\sum_{i=1}^{n} \dot{S}_{Qi} = -\frac{\dot{Q}}{T} - \frac{\dot{Q}_0}{T_0} \geq 0$$

$$-\dot{Q}_0 = T_0\left(\frac{\dot{Q}}{T} + \dot{S}_{irr}\right)$$

9.1 Carnot efficiency

$$\eta_{th} = 1 - \frac{T_0}{T} - \cancelto{0,\ \text{ideal}}{\frac{T_0\dot{S}_{irr}}{\dot{Q}}}$$

$$\eta_{th,max} = \eta_C(T, T_0) = 1 - \frac{T_0}{T}$$