I. Heat transfer

Symbology

Symbol	Quantity	Unit
λ , k	Thermal conductivity	$\mathrm{W/(m \cdot K)}$
$\dot{q}$	Heat flux	$\mathrm{W/m^2}$
$\dot{Q}$	Heat flow	W
α	Heat transfer coefficient	$\mathrm{W/(m^2 \cdot K)}$
a	Thermal diffusivity	$\mathrm{m^2/s}$
ν	Kinematic viscosity	$\mathrm{m^2/s}$

1. Types of heat transfer

1.1 Heat conduction

Occurs with all materials when there is a temperature gradient.

Steady-state conduction (1st Fourier's law):

$$\dot{q} = -\lambda \frac{\mathrm{d}T}{\mathrm{d}x}$$

Transient conduction (2nd Fourier's law 1D):

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial x^2}$$

1.2 Convective heat transfer

This occurs in moving fluids.

Newton's approach:

$$\dot{q} = \alpha \left(T_1 - T_2\right) = \frac{\lambda_{\text{fluid}}}{\delta_{\text{th}}} \left(T_1 - T_2\right)$$

1.3 Radiation

Bodies with a temperature above 0 K emit thermal radiation.

$$\dot{q}_{\text{rad}} = \epsilon \cdot \sigma \cdot \left(T^4 - T_\infty^4\right)$$

where σ is the Stefan-Boltzmann constant:
$\sigma = 5.67 \cdot 10^{-8} \ \left[\frac{W}{m^2 K^4}\right]$

2. Heat conduction and conductivity

2.1 Heat flux

$$\dot{q} = -\lambda \frac{\mathrm{d}T}{\mathrm{d}x} = k\Delta T = \frac{\lambda}{\delta}\Delta T$$

Procedure:

$$\vec{q} \int_0^\delta dx = -\lambda \int_{T_1}^{T_2} dT \Longrightarrow \dot{q} = \frac{\lambda}{\delta}\left(T_1 - T_2\right)$$

2.2 Heat flow

$$\dot{Q} = \frac{\Delta Q}{\Delta t} = \dot{q}A = kA\Delta T = \frac{\lambda}{\delta} A \Delta T$$

2.3 Thermal conduction k^{-1} and Thermal conductivity k

$$k_{\text{tot}}^{-1} = \sum_{i=1}^{n} \frac{\delta_i}{\lambda_i} \implies k_{\text{tot}} = \frac{1}{k_{\text{tot}}^{-1}}$$

3. Heat conduction through a hollow cylinder

$$\dot{Q} = -\lambda A \frac{\mathrm{d}T}{\mathrm{d}r} = -\lambda 2\pi r l \frac{\mathrm{d}T}{\mathrm{d}r}$$

$$\boxed{\dot{Q} = 2\pi l \cdot \frac{T_0 - T_1}{\frac{1}{\lambda} \ln \frac{r_1}{r_0}} = 2\pi r_0 l k \Delta T}$$

With multilayered walls:

$$\dot{Q} = 2\pi l \cdot \frac{T_0 - T_n}{\sum_{i=1}^{n} \frac{1}{\lambda_i} \ln \frac{r_i}{r_{i-1}}} = \boxed{2\pi r_0 l k_0 (T_0 - T_n)}$$

3.1 Thermal conductivity k

$$k_0 = \frac{1}{r_0 \left(\left(\frac{1}{\lambda_1} \ln \frac{r_1}{r_0}\right) + \left(\frac{1}{\lambda_2} \ln \frac{r_2}{r_1}\right) + \cdots + \left(\frac{1}{\lambda_n} \ln \frac{r_n}{r_{n-1}}\right)\right)}$$

$$k_0^{-1} = r_0 \sum_{i=1}^{n} \left(\frac{1}{\lambda_i} \ln \frac{r_i}{r_{i-1}}\right)$$

3.2 Thin-walled pipe approximated as a flat wall

$$\dot{Q}_T = \lambda A \frac{T_0 - T_1}{\delta} \quad ; \quad A = 2\pi l \left(r_0 + \frac{\delta}{2}\right)$$

$$\frac{\dot{Q}_T}{\dot{Q}_W} = \frac{\delta}{\left(r_0 + \frac{\delta}{2}\right) \ln\left(1 + \frac{\delta}{r_0}\right)} = \frac{\delta}{r_0} \frac{1}{\left(1 + \frac{\delta}{2r_0}\right) \ln\left(1 + \frac{\delta}{r_0}\right)}$$

4. Convective heat transfer

4.1 Reynolds number

It compares inertial to viscous forces.

$$\boxed{Re = \rho\, c \frac{L_{\text{char}}}{\eta} = c \frac{L_{\text{char}}}{\nu}}$$

$Re < 2300$: Laminar flow;
$Re \approx 2300$: Transition zone;
$Re > 2300$: Turbulent flow

4.2 Prandtl number

It compares momentum diffusivity to thermal diffusivity.
Relates the relative thickness of the velocity and thermal boundary layers.

$$Pr = \frac{\nu}{a} = \frac{\delta_{\text{fluid}}}{\delta_{\text{th}}} = \frac{\eta c_p}{\lambda} \quad ; \quad a = \frac{\lambda}{\rho c_p} \quad ; \quad \nu = \frac{\eta}{\rho}$$

4.3 Nusselt number

Measures convective heat transfer relative to pure conduction across a boundary layer and the heat transfer coefficient α.

$$Nu = \frac{\alpha \cdot L_{\text{char}}}{\lambda_{\text{fl}}} = \dot{q} \frac{L_{\text{char}}}{\lambda \Delta T_{\text{char}}}$$

$$\alpha = \frac{Nu \cdot \lambda}{L_{\text{char}}}$$

4.3.1 Forced convection

$$\boxed{Nu = c \cdot Re^m \cdot Pr^n}$$

where c is a constant.

For laminar flow over a flat plate:

$$Nu = 0.664 \cdot \sqrt{Re} \cdot \sqrt[3]{Pr}$$

For a flow in the transition zone (Tz):

$$Nu_{\text{Tz}} = \sqrt{Nu_{\text{lam}}^2 + Nu_{\text{turb}}^2}$$

4.4 Grashof number

It compares buoyancy to viscous forces.

$$Gr = g\beta\Delta T \frac{L^3}{\nu^2} \quad ; \quad g = 9.81 \ \text{N/kg}$$

$$\beta\Delta T = \frac{\rho_\infty - \rho_0}{\rho_\infty} = \frac{\Delta\rho}{\rho_\infty}$$

$$Gr = g \frac{\rho_\infty - \rho_0}{\rho_\infty} \frac{L^3}{\nu^2}$$

4.4.1 Natural convection

Peclet number

It compares advective to diffusive heat transport, i.e., the strength of convection relative to thermal diffusion.

$$Pe = RePr$$

Rayleigh number

It combines buoyancy driving and thermal diffusion, and it measures the intensity of natural convection.

$$Ra = GrPr$$

5. Heat exchanger

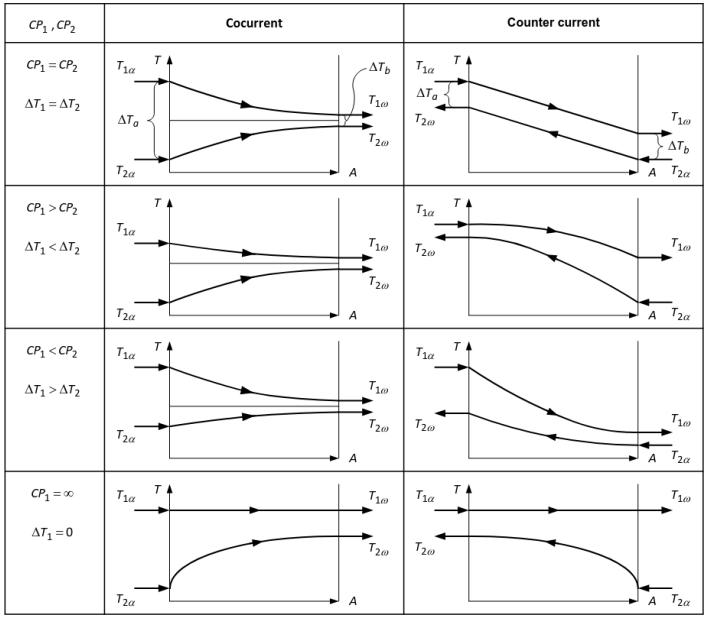
5.1 Energy balance

$$\dot{Q} = \dot{m} c_p \Delta\vartheta = CP\Delta\vartheta$$

$$\dot{Q}_1 = \dot{Q}_2$$

$$\dot{m}_1 c_{p1} \Delta\vartheta_1 = \dot{m}_2 c_{p2} \Delta\vartheta_2 \iff CP_1\Delta\vartheta_1 = CP_2\Delta\vartheta_2$$

5.2 Average temperature difference



5.2.1 Logarithmic mean temperature difference (LMTD)

$$\boxed{\Delta T_m = \frac{\Delta T_a - \Delta T_b}{\ln \dfrac{\Delta T_a}{\Delta T_b}}}$$

$$\Delta T_a = |\vartheta_{1,a} - \vartheta_{2,a}| \quad ; \quad \Delta T_b = |\vartheta_{1,b} - \vartheta_{2,b}|$$

5.2.2 Mean temperature heat flow

$$\boxed{\dot{Q} = kA\Delta T_m}$$

5.2.3 Heat capacity rate ratio (Capacity ratio)

It measures the relative ability of stream 1 vs. stream 2 to carry heat per kelvin:

$$CP_1\,(\vartheta_{1,\alpha} - \vartheta_{1,\omega}) = CP_2\,(\vartheta_{2\omega} - \vartheta_{2,\alpha})$$

$$R_1 = \frac{CP_1}{CP_2} = \frac{\vartheta_{2,\omega} - \vartheta_{2,\alpha}}{\vartheta_{1,\alpha} - \vartheta_{1,\omega}} = \frac{\Delta\vartheta_2}{\Delta\vartheta_1}$$

5.3 Co-current vs countercurrent flow heat exchanger

$$\dot{Q} = kA_{\text{co}}\Delta T_{m,\text{co}} = kA_{\text{count}}\Delta T_{m,\text{count}}$$

$$\frac{A_{\text{co}}}{A_{\text{count}}} = \frac{\Delta T_{m,\text{count}}}{\Delta T_{m,\text{co}}}$$

$$\frac{A_{\text{co}}}{A_{\text{count}}} = \frac{\dot{Q}_{\text{co}}}{\dot{Q}_{\text{count}}} \cdot \frac{\Delta T_{m,\text{count}}}{\Delta T_{m,\text{co}}}, \quad \text{if } \dot{Q}_1 \neq \dot{Q}_2$$

II. The 2nd law of thermodynamics

6. Recap of the 1st law

6.1 Closed systems

$$q_{12} + w_{12} = u_2 - u_1$$

6.2 Open systems

$$q_{12} + w_{12} = h_2 - h_1 + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)$$

$$w_{t12} = \int v\,dp + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right) + j_{12}$$

$$q_{12} + \int v\,dp + j_{12} = h_2 - h_1 \overset{\text{ideal gas}}{=\!=} c_p\left(T_2 - T_1\right)$$

7. The 2nd law

7.1 Statements

$$\boxed{\text{Energy} = \text{Exergy} + \text{Anergy}}$$

Processes	Reversible	Irreversible
Exergy	Conserved	Reduced → Loss
Entropy	Constant	Increases → Production

7.2 Entropy

Entropy S: extensive state variable [J/K]

Specific entropy s: intensive state variable [J/kgK]

Entropy flow $\dot{S}$: entropy over time [W/K]

8. Entropy balance equation

$$\boxed{dS = dS_Q + dS_m + dS_{irr}}$$

8.1 Entropy balance terms

- Heat transfer across the system boundary:

$$dS_Q = \frac{dQ}{T}$$

- Mass flow across the system boundary:

$$dS_{\text{m}} = \dot{m}\,s\,dt \quad ; \quad \dot{S}_m = \dot{m}\,s$$

- Irreversible processes inside the system:

$$\dot{S}_{irr}(t)\begin{cases} dS_{irr} > 0 & \text{irreversible processes} \\ dS_{irr} = 0 & \text{reversible processes} \\ dS_{irr} < 0 & \text{impossible} \end{cases}$$

8.2 Entropy balance equation for closed systems

$$dS = dS_Q + \cancel{dS_m}^{\,0} + dS_{irr}$$

$$dS = \dot{S}_Q\,dt + \dot{S}_{irr}\,dt = \left(\frac{\dot{Q}}{T} + \dot{S}_{irr}\right)dt$$

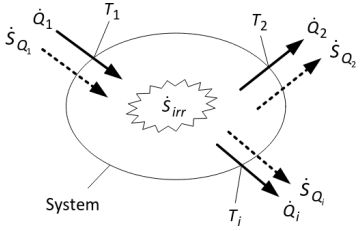
8.2.1 Closed system entropy flow balance equation

$$\frac{dS}{dt} = \sum_i \dot{S}_{Qi} + \dot{S}_{irr} \quad ; \quad \dot{S}_{irr} \geq 0$$

For steady-state case:

$$\dot{S}_{irr} = -\sum_i \dot{S}_{Qi} = -\sum_i \frac{\dot{Q}_i}{T_i} \geq 0$$

8.2.2 Visual representation



$$\boxed{\sum_i \dot{S}_{Qi} = \sum_i \frac{\dot{Q}_i}{T_i} = -\dot{S}_{irr}}$$

8.3 Entropy balance equation for open systems

$$\frac{dS}{dt} = \sum_\alpha \dot{m}_\alpha s_\alpha - \sum_\omega \dot{m}_\omega s_\omega + \sum \dot{S}_Q(t) + \dot{S}_{irr}(t)$$

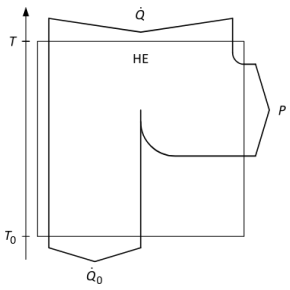
8.3.1 Steady-state flow processes

$$\dot{S}_{irr} = \sum_\omega \dot{m}_\omega s_\omega - \sum_\alpha \dot{m}_\alpha s_\alpha - \sum \dot{S}_Q \geq 0$$

8.3.2 Steady-state and adiabatic flow processes

$$\dot{S}_{irr} = \sum_\omega \dot{m}_\omega s_\omega - \sum_\alpha \dot{m}_\alpha s_\alpha \geq 0$$

9. The thermal engine



$$\dot{Q} + \dot{Q}_0 + P = 0$$

$$\eta_{th} = \frac{-P}{\dot{Q}} = 1 - \frac{\left|\dot{Q}_0\right|}{\dot{Q}}$$

$$\dot{S}_{irr} = -\sum_{i=1}^{n} \dot{S}_{Qi} = -\frac{\dot{Q}}{T} - \frac{\dot{Q}_0}{T_0} \geq 0$$

$$-\dot{Q}_0 = T_0\left(\frac{\dot{Q}}{T} + \dot{S}_{irr}\right)$$

9.1 Carnot efficiency

$$\eta_{th} = 1 - \frac{T_0}{T} - \frac{T_0\cancel{\dot{S}_{irr}}^{\,0,\ \text{ideal}}}{\dot{Q}}$$

$$\eta_{th,max} = \eta_C(T, T_0) = 1 - \frac{T_0}{T}$$

10. Entropy S as a state variable

$$ds = \frac{dq + dj}{T} = \frac{du + p\,dv}{T} = \frac{dh - v\,dp}{T}$$
$$T\,ds = dq + dj \Longrightarrow T\,(s_2 - s_1) = q_{12} + j_{12}$$

General form: $\int_1^2 T\,ds = q_{12} + j_{12}$

10.1 Reversible processes

10.1.1 Clausius entropy relation

$$\boxed{\delta q_{\text{rev}} = T\,ds \Longleftrightarrow \delta Q_{\text{rev}} = T\,dS}$$

10.1.2 Work W and heat Q

$$w_{p,\,rev} = \int v\,dp$$
$$Q_{rev} = \int T\,dS$$

10.2 Dissipation j as a state variable

10.2.1 Dissipation

$$w_{t12} = w_{p12} + j_{12} \Longleftrightarrow w_{p12} = w_{t12} - j_{12}$$
$$dq_{rev} + dw_{v,\,rev} = du$$
$$T\,ds = dq + T\,ds_{irr} = dq_{rev} + dj$$
$$\int_1^2 T\,ds = q_{12} + j_{12}$$

10.2.2 For closed systems

$$ds = \frac{du + p\,dv}{T} = \frac{c_v\,dT}{T} + \frac{R_i\,dv}{v}$$

$$\boxed{s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R_i \ln\frac{v_2}{v_1}}$$

10.2.3 Work change

All the works can be calculated with a temperature change:

$$q_{12} + w_{t12} = h_2 - h_1 + \frac{c_2^2 - c_1^2}{2} + g\,(z_2 - z_1)$$
$$q_{12} + \int v\,dp + j_{12} = h_2 - h_1 = c_p\,(T_2 - T_1)$$
$$\int T\,ds = q_{12} + j_{12} = \boxed{\int T\,ds + \int v\,dp = h_2 - h_1}$$
$$\boxed{T\,ds + v\,dp = h_2 - h_1 \Longleftrightarrow T\,ds - p\,dv = u_2 - u_1}$$
$$T\,ds + v\,dp = dh\,T\,ds - p\,dv = du$$

10.3 Reversible state changes with perfect gases

10.3.1 Open system

$$q_{12} + j_{12} + w_{p,rev} = h_2 - h_1$$
$$\int_1^2 T\,ds + \int_1^2 v\,dp = c_p\,(T_2 - T_1)$$

10.3.2 Closed system

$$q_{12} + j_{12} + w_{v,rev} = u_2 - u_1$$
$$\int_1^2 T\,ds - \int_1^2 p\,dv = c_v\,(T_2 - T_1)$$

10.4 Polytropic state changes

$$T\,ds = dq + dj = du + p\,dv = dh - v\,dp$$
$$du + p\,dv = c_v\,dT + p\,dv \quad ; \quad dh - v\,dp = c_p\,dT - v\,dp$$
$$dw_v = c_v\frac{k-1}{n-1}\,dT = -p\,dv$$

For the change in entropy of the polytropic state change:

$$ds = c_v\frac{n-k}{n-1}\frac{dT}{T}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\boxed{s_2 - s_1 = c_v\frac{n-k}{n-1}\ln\frac{T_2}{T_1}} \quad n \neq 1$$

$$\boxed{s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R_i \ln\frac{v_2}{v_1} = c_p \ln\frac{T_2}{T_1} - R_i \ln\frac{p_2}{p_1}}$$

Compressor with dissipation: $n > k$
Turbine with dissipation: $n < k$

11. Exergy and anergy

Exergy: useful part of energy that can be completely converted into work under given ambient conditions.

Anergy: remaining part of energy that cannot be converted into work under given ambient conditions.

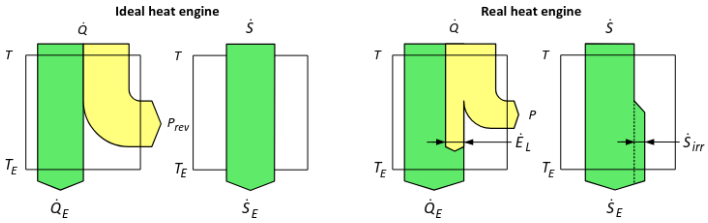
$$\boxed{e = ex + an}$$

11.1 Statements of the 2nd LT

I. In all irreversible processes exergy is converted into anergy
II. Only in reversible processes exergy remains constant
III. It is impossible to convert anergy into exergy

11.2 Exergy

11.2.1 Exergy balance / Exergy loss



$$\dot{E}_L = \sum_\alpha \dot{E}_\alpha - \sum_\omega |\dot{E}_\omega| = \dot{Q}\frac{T_A - T_E}{T_A} + \left(-\dot{Q}\frac{T_B - T_E}{T_B}\right)$$

$$\boxed{\dot{E}_L = T_E\dot{S}_{irr} = T_E\frac{T_A - T_B}{T_A T_B}\dot{Q}}$$

$$\dot{S}_{irr} = \sum_\omega \dot{m}_\omega s_\omega - \sum_\alpha \dot{m}_\alpha s_\alpha - \sum \dot{S}_Q \geq 0, \quad d\dot{S}_Q = \frac{d\dot{Q}}{T}$$

11.2.2 Exergy of heat

$$\dot{E}_Q = P_{rev} = \left(1 - \frac{T_E}{T}\right)\dot{Q} = \eta_C\dot{Q}$$

11.2.3 Exergy of material flows (without mixtures)

$$e = h - h_E - T_E\,(s - s_E) + \frac{c^2}{2} + gz + q\left(1 - \frac{T_E}{T}\right)$$

$$\boxed{e_2 - e_1 = h_2 - h_1 - T_E\,(s_2 - s_1)}$$

Perfect gases

$$e = c_p\left(T - T_E - T_E \ln\frac{T}{T_E}\right) + R_i T_E \ln\frac{p}{p_E}$$

Incompressible liquids (isobaric)

$$e = c_p\left(T - T_E - T_E \ln\frac{T}{T_E}\right)$$

Isobaric for $T < T_E$ (cooling)

$$e = c_p\left(T_E - T - T_E \ln\frac{T_E}{T}\right)$$

III. Steady-state flow processes

12. Adiabatic working cycles

12.1 Compressor

12.1.1 1st law

$$w_{t12} + q_{12} = h_2 - h_1 + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)$$

$$w_{t12} + q_{12} = c_p\left(T_2 - T_1\right) + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)$$

$$P_{12} + \dot{Q}_{12} = \dot{m}\left(w_{t12} + q_{12}\right)$$

12.1.2 Isentropic compression efficiency factor

$$\eta_s = \frac{\text{isentropic work}}{\text{real polytropic work}}$$

$$\eta_s = \frac{\Delta h_s}{\Delta h} = \frac{h_{2,s} - h_1}{h_2 - h_1} = \frac{c_{p,m}\left(T_{2,s} - T_1\right)}{c_{p,m}\left(T_2 - T_1\right)} = \frac{T_{2,s} - T_1}{T_2 - T_1} \leq 1$$

12.2 Turbine

12.2.1 1st law

$$w_{t12} + q_{12} = h_2 - h_1 + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)$$

$$w_{t12} + q_{12} = c_p\left(T_2 - T_1\right) + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)$$

$$P_{12} = \dot{m} w_{t12} \iff -P_{12} = \dot{m}\left|w_{t12}\right|$$

12.2.2 Isentropic turbine efficiency

$$\eta_s = \frac{\text{real polytropic work}}{\text{isentropic work}}$$

$$\eta_s = \frac{-\Delta h}{-\Delta h_s} = \frac{h_1 - h_2}{h_1 - h_{2,s}} = \frac{T_1 - T_2}{T_1 - T_{2,s}}$$

13. Adiabatic flow processes without work

13.1 Throttle / Expansion valve (isenthalpic)

$$h_1 = h_2 \Longrightarrow j_{12} = -\int_1^2 v\, dp = -R_i T \ln\frac{p_2}{p_1}$$

13.2 Nozzle

$$h_2^0 = h_1^0 \Longrightarrow h_2 + \frac{1}{2}c_2^2 = h_1 + \frac{1}{2}c_1^2$$

$$c_2 = \sqrt{2\left(h_1 - h_2\right) + c_1^2}$$

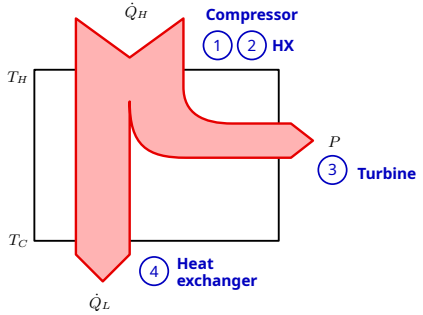
where h^0 is the total enthalpy: $h^0 = h + \frac{c^2}{2}$

IV. Thermodynamics cycles

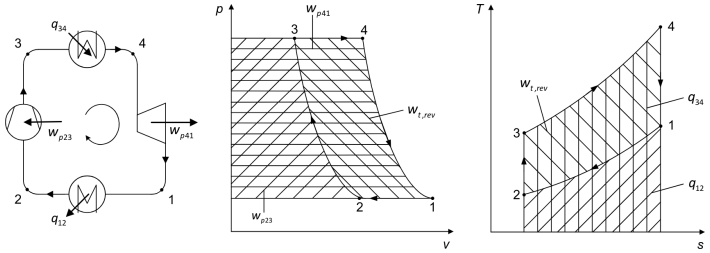
14. Reversible cycle work

Total cycle / sum of all sub-processes:

$$w_{t,\,rev} = \int_1^2 v\, dp + \int_2^3 v\, dp + \ldots + \int_n^1 v\, dp = \oint v\, dp = \sum_{i=1}^{n} w_{pi}$$



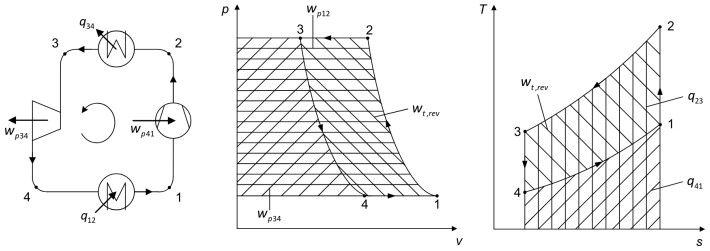
14.1 Clockwise cycle



$$w_{t,rev} = w_{p,23} + w_{p,41} < 0$$

$$\left|w_{t,rev}\right| = \left|q_{34}\right| - \left|q_{12}\right|$$

14.2 Counterclockwise cycle

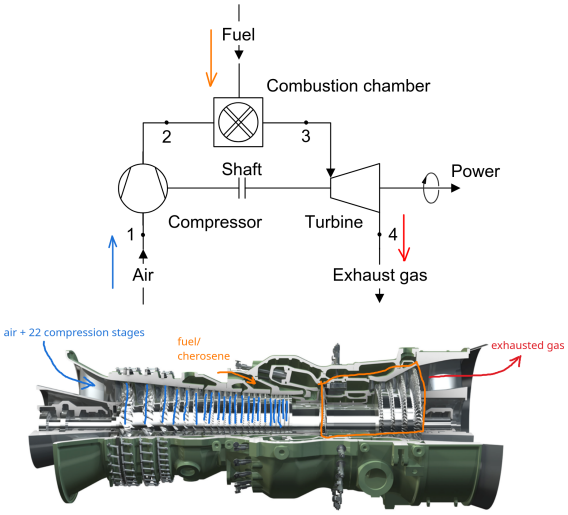


$$w_{t,rev} = w_{p,12} + w_{p,34} < 0$$

$$\left|w_{t,rev}\right| = \left|q_{23}\right| - \left|q_{41}\right|$$

V. Thermal systems with gas turbines

15. Turbine



Power balance:

$$\dot{Q}_{23} + \dot{Q}_{41} + P_C = 0$$

15.1 Joule process

$$\pi = \frac{p_2}{p_1} = \frac{p_3}{p_4}$$

$$-w_C = q_\alpha + q_\omega = q_{23} + q_{41} = c_p\left(T_3 - T_2 + T_1 - T_4\right)$$

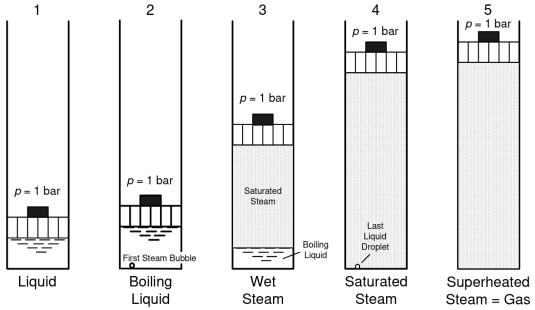
$$\left|w_C\right| = c_p\left(T_3 - T_2\right)\left(1 - \frac{T_4 - T_1}{T_3 - T_2}\right)$$

$$\frac{T_4 - T_1}{T_3 - T_2} = \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}} = \left(\frac{1}{\pi}\right)^{\frac{k-1}{k}}$$

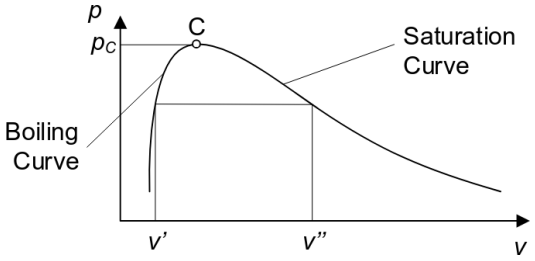
$$\eta_J = \frac{\left|w_C\right|}{q_\alpha} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{1}{\pi}\right)^{\frac{k-1}{k}}$$

VI. Thermal plants with steam turbines

16. Types of steam



17. Boundary curves of the two phase region



17.1 Steam tables

$$v = (1-x)v' + xv'' = v' + x(v'' - v')$$

$$h = (1-x)h' + xh'' = h' + x\Delta h_v$$

$$s = (1-x)s' + xs'' = s' + x\left(\frac{\Delta h_v}{T}\right)$$

17.1.1 Steam content

$$x = \frac{v - v'}{v'' - v'}$$

$$x = \frac{h - h'}{h'' - h'} = \frac{h - h'}{\Delta h_v}$$

$$x = \frac{s - s'}{s'' - s'} = (s - s')\frac{T}{\Delta h_v}$$

17.1.2 Subcooling / Condenser enthalpy

$$h_1 = h'([p,t,s]) - c_p \Delta T_{\text{subcooling}}$$

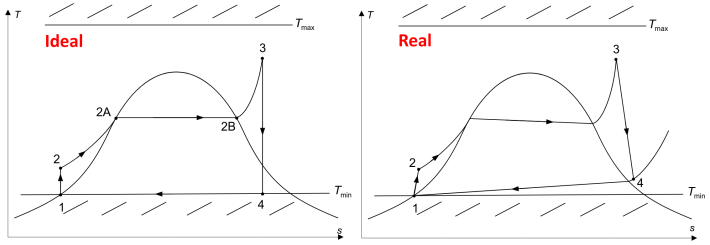
$$\dot{Q}_{\text{heating}} = \dot{m} \cdot c_p \Delta T_{\text{heating}}$$

17.2 Water pump

$$P_{\text{el}} \cdot \eta_p = \dot{m} \cdot c_p \cdot \Delta T$$

18. Steam power stations

18.1 Rankine cycle



$w_{t12} = h_2 - h_1$ (pump) $\quad |w_{t34}| = h_3 - h_4$ (turbine)

$q_{23} = h_3 - h_2$ (boiler) $\quad |q_{41}| = h_4 - h_1$ (condenser)

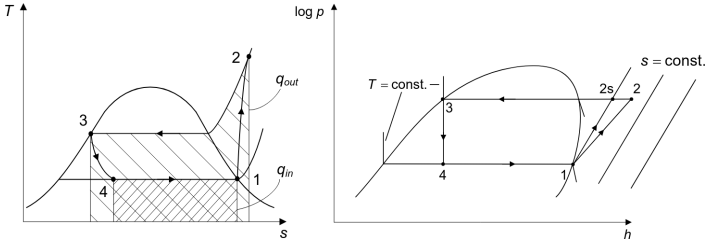
$$\eta_{th} = \frac{(h_3 - h_4) \overbrace{-(h_2 - h_1)}^{\text{pump work}}}{h_3 - h_2} \quad ; \quad \eta_s = \frac{h_3 - h_4}{h_3 - h_{4,s}} \leq 1$$

18.1.1 Generator

$$P_G = \dot{m}_D \cdot (h_3 - h_4) \cdot \prod_i \eta_i$$

VII. Heat pumps and Chillers

19. T-s and log p-h diagrams



$w_{t,12} = h_2 - h_1$ (compressor) $\quad h_3 = h_4$ (expansion valve)

$q_{23} = h_2 - h_3$ (condenser) $\quad q_{41} = h_1 - h_4$ (evaporator)

19.1 Isentropic efficiency (compressor)

$$\eta_s = \frac{h_{2,s} - h_1}{h_2 - h_1} \leq 1$$

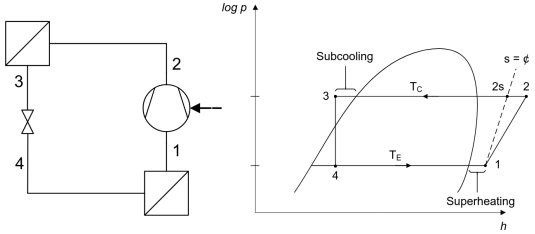
20. Heat pumps (HP)

20.1 Inner coefficient of performance

$$\varepsilon_{\text{HP}} = \frac{|q_{23}|}{w_{12}} = \frac{\text{heat}}{\text{technical work}} = \frac{h_2 - h_3}{h_2 - h_1} = \varepsilon_R + 1$$

$$\text{Carnot: } \frac{1}{\eta_C} = \varepsilon_{HP,C} = \frac{T_C}{T_C - T_E} = \frac{T_C}{\Delta T_{\text{lift}}} = \varepsilon_{R,C} + 1$$

20.2 Outer coefficient of performance (COP)



$$\text{COP}_{\text{HP}} = \frac{|\dot{Q}_\omega|}{P_{el}} = \frac{|\dot{Q}|}{P_{el}} = \eta_{\text{mech}} \cdot \varepsilon_{\text{HP}}$$

$$\varepsilon_{HP} > COP_{HP} \quad ; \quad \zeta_{HP} = \frac{COP_{HP}}{\varepsilon_{HP,C}}$$

20.3 Real inner COP

Assumptions: $\zeta = 0.5,\ \Delta T_e = 10K,\ \Delta T_c = 5K$

$$\varepsilon_{HP,C} = \frac{\vartheta_{\text{req}} - \Delta T_c}{(\vartheta_{\text{env}} - \Delta T_c) - (\vartheta_{\text{req}} - \Delta T_e)}$$

$$\varepsilon_{\text{HP}} = \zeta_{HP} \cdot \varepsilon_{HP,C}$$

21. Chillers / refrigerators (R)

21.1 Inner coefficient of performance

$$\varepsilon_R = \frac{|q_{41}|}{w_{12}} = \frac{h_1 - h_4}{h_2 - h_1} = \varepsilon_{\text{HP}} - 1$$

$$\text{Carnot: } \varepsilon_{R,C} = \frac{T_E}{T_C - T_E} = \frac{T_E}{\Delta T_{\text{lift}}} = \varepsilon_{HP,C} - 1$$

21.2 Outer coefficient of performance (COP)

$$EER = COP_R = \frac{\dot{Q}_{in}}{P_{el}} = \frac{\dot{Q}_e}{P_{el}} = \eta_{\text{mech}} \cdot \varepsilon_R$$

$$\varepsilon_R > EER \quad ; \quad \zeta_R = \frac{EER}{\varepsilon_{R,C}}$$

21.3 Real inner COP

Assumptions: $\zeta = 0.5,\ \Delta T_e = 10K,\ \Delta T_c = 5K$

$$\varepsilon_{R,C} = \frac{\vartheta_{\text{req}} - \Delta T_e}{(\vartheta_{\text{env}} - \Delta T_c) - (\vartheta_{\text{req}} - \Delta T_e)}$$

$$\varepsilon_R = \zeta_R \cdot \varepsilon_{R,C}$$

VIII. Data interpolation and extrapolation

22. Interpolation

For any property X, with $X \in \{v, h, s\}$, the general linear interpolation formula is:

$$X = X_1 + \frac{x - x_1}{x_2 - x_1}(X_2 - X_1)$$

where x can be pressure, temperature, or another independent variable.

22.1 Temperature interpolation at constant pressure

If the pressure is already available in the table, but the required temperature lies between two tabulated temperatures ϑ_1 and ϑ_2, then:

$$v = v_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(v_2 - v_1)$$

$$h = h_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(h_2 - h_1)$$

$$s = s_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(s_2 - s_1)$$

22.2 Pressure interpolation at constant temperature

If the temperature is already available in the table, but the required pressure lies between two tabulated pressures p_1 and p_2, then:

$$v = v_{p_1} + \frac{p - p_1}{p_2 - p_1}(v_{p_2} - v_{p_1})$$

$$h = h_{p_1} + \frac{p - p_1}{p_2 - p_1}(h_{p_2} - h_{p_1})$$

$$s = s_{p_1} + \frac{p - p_1}{p_2 - p_1}(s_{p_2} - s_{p_1})$$

22.3 Double interpolation in pressure and temperature

If both pressure and temperature are between table values, first interpolate with respect to temperature at p_1 and p_2, then interpolate with respect to pressure.

Let: $p_1 < p < p_2 \quad ; \quad \vartheta_1 < \vartheta < \vartheta_2$

For any property $X \in \{v, h, s\}$:

$$X_{p_1} = X_{11} + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(X_{12} - X_{11})$$

$$X_{p_2} = X_{21} + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(X_{22} - X_{21})$$

$$X = X_{p_1} + \frac{p - p_1}{p_2 - p_1}(X_{p_2} - X_{p_1})$$

22.4 Saturated steam values

For saturated steam properties at the top of the steam table, the same pressure interpolation formula can be used:

$$X'' = X_1'' + \frac{p - p_1}{p_2 - p_1}(X_2'' - X_1'')$$

where:

$$X'' \in \{v'', h'', s''\}$$

23. Extrapolation

For any property X, with $X \in \{v, h, s\}$:

$$X = X_1 + \frac{x - x_1}{x_2 - x_1}(X_2 - X_1)$$

where x is outside the interval $[x_1, x_2]$.

23.1 Pressure extrapolation at constant temperature

If the required pressure is outside the tabulated pressure range, use the two nearest available tabulated pressures. For any property $X \in \{v, h, s\}$:

$$X = X_{p_1} + \frac{p - p_1}{p_2 - p_1}(X_{p_2} - X_{p_1})$$

where p is outside the interval $[p_1, p_2]$.

For the individual steam-table properties:

$$v = v_{p_1} + \frac{p - p_1}{p_2 - p_1}(v_{p_2} - v_{p_1})$$

$$h = h_{p_1} + \frac{p - p_1}{p_2 - p_1}(h_{p_2} - h_{p_1})$$

$$s = s_{p_1} + \frac{p - p_1}{p_2 - p_1}(s_{p_2} - s_{p_1})$$

23.2 Temperature extrapolation at constant pressure

If the required temperature is outside the tabulated temperature range, use the two nearest available temperatures. For any property $X \in \{v, h, s\}$:

$$X = X_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(X_2 - X_1)$$

For the individual properties:

$$v = v_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(v_2 - v_1)$$

$$h = h_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(h_2 - h_1)$$

$$s = s_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(s_2 - s_1)$$

23.3 Saturated steam extrapolation

For saturated steam properties outside the tabulated pressure range:

$$X'' = X_1'' + \frac{p - p_1}{p_2 - p_1}(X_2'' - X_1'')$$

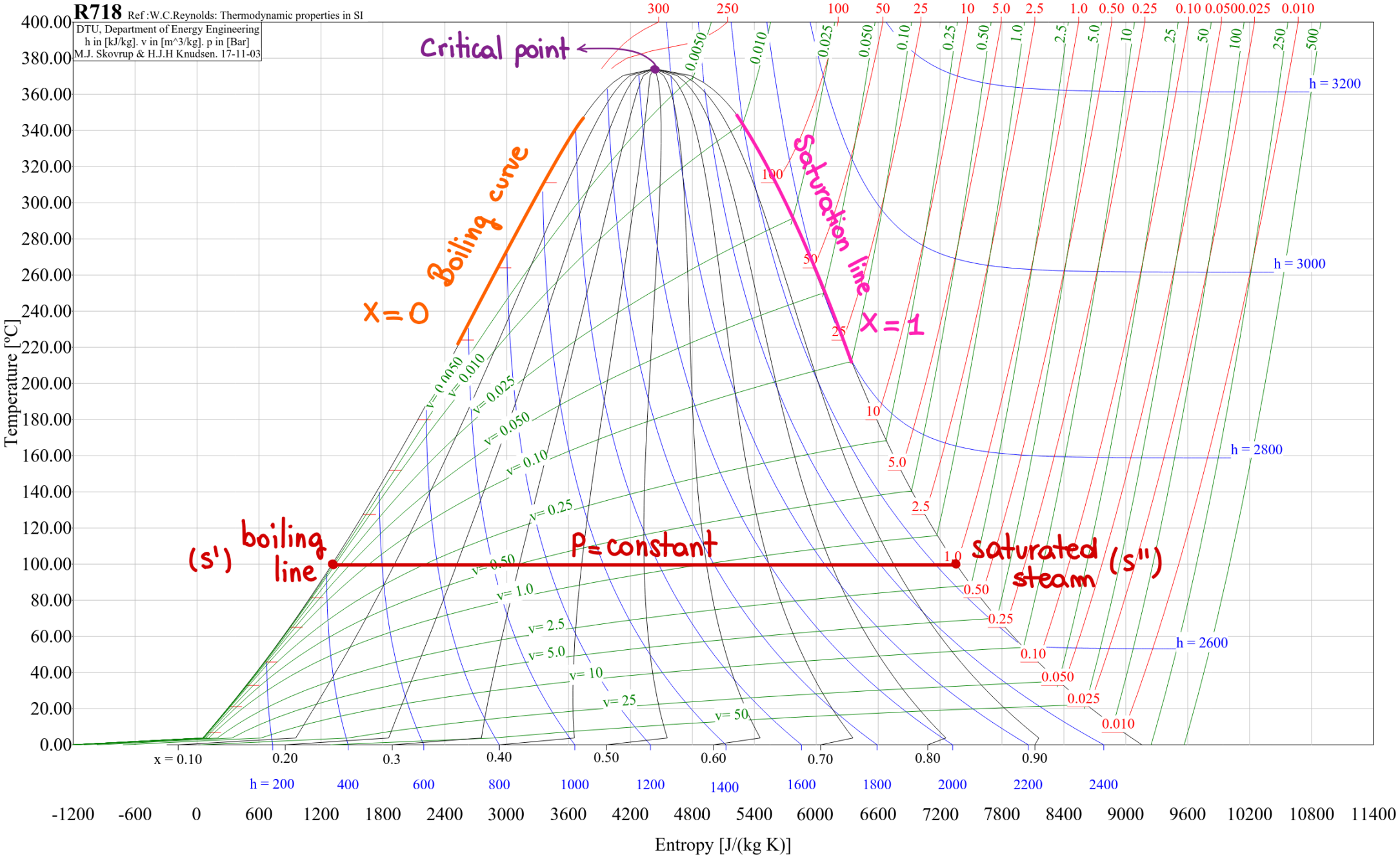
where:

$$X'' \in \{v'', h'', s''\}$$

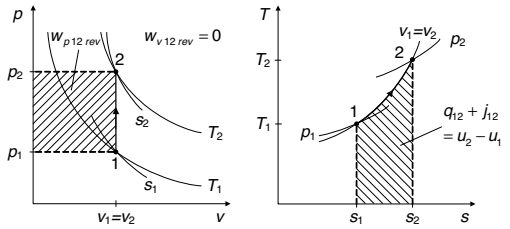
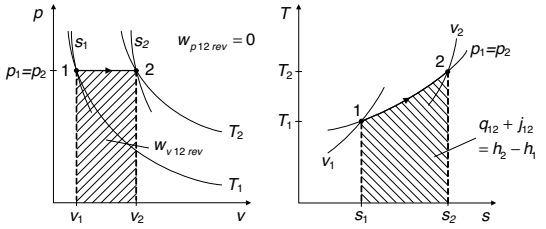
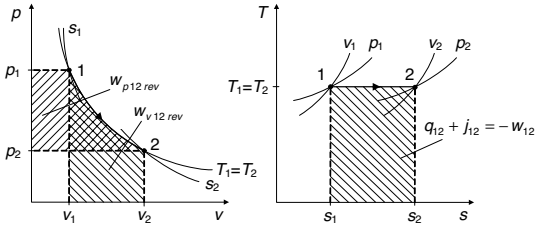
For example, using 25 bar and 50 bar to estimate the value at 100 bar:

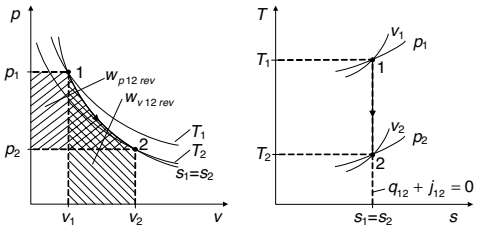
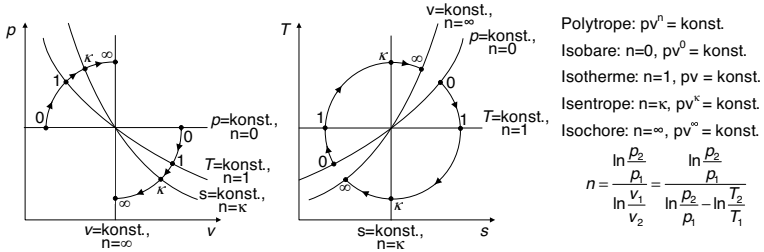
$$X_{100}'' = 3X_{50}'' - 2X_{25}''$$

A. T-s diagram


R718 Ref :W.C.Reynolds: Thermodynamic properties in SI
DTU, Department of Energy Engineering
h in [kJ/kg]. v in [m^3/kg]. p in [Bar]
M.J. Skovrup & H.J.H Knudsen. 17-11-03
Critical point
Boiling curve
X = 0
Saturation line
X = 1
boiling line
(s')
P = constant
saturated steam (s'')
h = 3200
h = 3000
h = 2800
h = 2600
v= 0.0050
v= 0.010
v= 0.025
v= 0.050
v= 0.10
v= 0.25
v= 0.50
v= 1.0
v= 2.5
v= 5.0
v= 10
v= 25
v= 50
Temperature [ºC]
Entropy [J/(kg K)]

Special state changes with perfect gases (without consideration of kinetic and potential energies)

Ideal gas	**v = constant: Isochoric state change**	**p = constant: Isobaric state change**	**T = constant: Isothermal state change**
Therm. state equation $p\,V = n\,R\,T$ $R = 8.314\,\mathrm{J/mol\,K}$ $p\,V = m\,R_i\,T$ $R_i = \dfrac{R}{M_i}$ $p\,v = R_i\,T, \quad \rho = \dfrac{p}{R_i\,T}$ $R_i = c_p - c_v, \quad \kappa = \dfrac{c_p}{c_v}$ $c_v = \dfrac{R_i}{\kappa - 1}, \quad c_p = \dfrac{\kappa\,R_i}{\kappa - 1}$	 $v = \text{const.}: \quad \dfrac{p_1}{T_1} = \dfrac{p_2}{T_2} = \dfrac{p}{T} = \text{const.}$ $w_{v\,12\,rev} = 0 \quad (\mathrm{d}v = 0)$ $w_{p\,12\,rev} = v\,(p_2 - p_1) = \dfrac{p_2 - p_1}{\rho}$	 $p = \text{const.}: \quad \dfrac{v_1}{T_1} = \dfrac{v_2}{T_2} = \dfrac{v}{T} = \text{const.}$ $w_{v\,12\,rev} = -p\,(v_2 - v_1) = -R_i\,(T_2 - T_1)$ $w_{p\,12\,rev} = 0 \quad (\mathrm{d}p = 0)$	 $T = \text{const.}: \quad p_1\,v_1 = p_2\,v_2 = p\,v = \text{const.}$ $w_{v\,12\,rev} = -p_1\,v_1\,\ln\dfrac{v_2}{v_1} = p_1\,v_1\,\ln\dfrac{p_2}{p_1} = R_i\,T\,\ln\dfrac{p_2}{p_1}$ $w_{p\,12\,rev} = w_{v\,12\,rev}$ [J] [J/kg]
1st LT closed system $q_{12} + w_{12} = u_2 - u_1$ Rev. volume change work: $w_{v\,12\,rev} = -\displaystyle\int_1^2 p\,\mathrm{d}v$ $w_{v\,12} = w_{v\,12\,rev} + j_{12}$ Overall work: $w_{12} = w_{v\,12} + w_{W\,12} + w_{el\,12}$	$q_{12} + w_{12} = u_2 - u_1$ Rev. SC: $w_{12} = w_{v\,12\,rev} = 0 \quad (\mathrm{d}v = 0)$ $q_{12\,rev} = u_2 - u_1 = c_v\,(T_2 - T_1)$	$q_{12} + w_{12} = u_2 - u_1$ Rev. SC: $w_{12} = w_{v\,12\,rev} = -p\,(v_2 - v_1)$ $q_{12\,rev} = u_2 - u_1 + p\,(v_2 - v_1) = h_2 - h_1$ $q_{12\,rev} = c_p\,(T_2 - T_1)$	$q_{12} + w_{12} = u_2 - u_1 = c_v\,(T_2 - T_1) = 0$ $q_{12} = -w_{12}$ Rev. SC: $q_{12\,rev} = -w_{v\,12\,rev}$ $q_{12\,rev} = p_1\,v_1\,\ln\dfrac{v_2}{v_1} = -p_1\,v_1\,\ln\dfrac{p_2}{p_1} = -R_i\,T\,\ln\dfrac{p_2}{p_1}$
1st LT open system $q_{12} + w_{t\,12} = h_2 - h_1$ Rev. pressure change work: $w_{p\,12\,rev} = \displaystyle\int_1^2 v\,\mathrm{d}p$ Technical work: $w_{t\,12} = \displaystyle\int_1^2 v\,\mathrm{d}p + j_{12}$	$q_{12} + w_{t\,12} = h_2 - h_1$ Rev. SC: $w_{t\,12} = w_{p\,12\,rev} = v\,(p_2 - p_1)$ $q_{12\,rev} = h_2 - h_1 - v\,(p_2 - p_1) = u_2 - u_1$ $q_{12\,rev} = c_v\,(T_2 - T_1)$	$q_{12} + w_{t\,12} = h_2 - h_1$ Rev. SC: $w_{t\,12} = w_{p\,12\,rev} = 0 \quad (\mathrm{d}p = 0)$ $q_{12\,rev} = h_2 - h_1 = c_p\,(T_2 - T_1)$	$q_{12} + w_{t\,12} = h_2 - h_1 = c_p\,(T_2 - T_1) = 0$ $q_{12} = -w_{t\,12}$ Rev. SC: $q_{12\,rev} = -w_{p\,12\,rev}$ $q_{12\,rev} = p_1\,v_1\,\ln\dfrac{v_2}{v_1} = -p_1\,v_1\,\ln\dfrac{p_2}{p_1} = -R_i\,T\,\ln\dfrac{p_2}{p_1}$
2nd LT / entropy $\mathrm{d}s = \dfrac{\mathrm{d}q + \mathrm{d}j}{T}$ $\mathrm{d}s = \dfrac{\mathrm{d}u + p\,\mathrm{d}v}{T} = \dfrac{\mathrm{d}h - v\,\mathrm{d}p}{T}$	$\mathrm{d}s = \dfrac{\mathrm{d}u}{T} = \dfrac{c_v\,\mathrm{d}T}{T} \Rightarrow s_2 - s_1 = c_v\,\ln\dfrac{T_2}{T_1}$	$\mathrm{d}s = \dfrac{\mathrm{d}h}{T} = \dfrac{c_p\,\mathrm{d}T}{T} \Rightarrow s_2 - s_1 = c_p\,\ln\dfrac{T_2}{T_1}$	$\mathrm{d}s = \dfrac{p\,\mathrm{d}v}{T} = \dfrac{R_i\,\mathrm{d}v}{v} \Rightarrow s_2 - s_1 = R_i\,\ln\dfrac{v_2}{v_1}$ $\mathrm{d}s = -\dfrac{v\,\mathrm{d}p}{T} = -\dfrac{R_i\,\mathrm{d}p}{p} \Rightarrow s_2 - s_1 = -R_i\,\ln\dfrac{p_2}{p_1}$

Ideal gas	s = **constant: Isentropic state change**	n = **constant: Polytropic state change**
Therm. state change $p\,V = n\,R\,T$ $R = 8.314\,\mathrm{J/mol\,K}$ $p\,V = m\,R_i\,T$ $R_i = \dfrac{R}{M_i}$ $p\,v = R_i\,T, \quad \rho = \dfrac{p}{R_i\,T}$ $R_i = c_p - c_v, \quad \kappa = \dfrac{c_p}{c_v}$ $c_v = \dfrac{R_i}{\kappa - 1}, \quad c_p = \dfrac{\kappa\,R_i}{\kappa - 1}$	 $s = \text{const}: \quad p_1\,v_1^{\kappa} = p_2\,v_2^{\kappa} = \text{const.} \quad \Rightarrow \quad \dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} = \left(\dfrac{v_1}{v_2}\right)^{\kappa-1}$ $w_{v\,12\,rev} = \dfrac{p_1\,v_1}{\kappa - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{\kappa-1} - 1\right] = \text{etc. (replace)}$ $w_{p\,12\,rev} = \kappa\,w_{v\,12\,rev} = \kappa\,\dfrac{p_1\,v_1}{\kappa - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{\kappa-1} - 1\right] = \text{etc. (replace)}$	 $n = \dfrac{\ln\frac{p_2}{p_1}}{\ln\frac{v_1}{v_2}} = \dfrac{\ln\frac{p_2}{p_1}}{\ln\frac{p_2}{p_1} - \ln\frac{T_2}{T_1}}$ $n = \text{const}: \quad p_1\,v_1^{n} = p_2\,v_2^{n} = \text{const.} \quad \Rightarrow \quad \dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\dfrac{v_1}{v_2}\right)^{n-1}$ $w_{v\,12} = \dfrac{p_1\,v_1}{n - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{n-1} - 1\right] = \text{etc. (replace, } n \neq 1)$ $w_{p\,12} = n\,w_{v\,12} = n\,\dfrac{p_1\,v_1}{n - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{n-1} - 1\right] = \text{etc. (replace, } n \neq 1)$
1st LT closed system $q_{12} + w_{12} = u_2 - u_1$ Rev. volume change work: $w_{v\,12\,rev} = -\int_1^2 p\,\mathrm{d}v$ $w_{v\,12} = w_{v\,12\,rev} + j_{12}$ Overall work: $w_{12} = w_{v\,12} + w_{W\,12} + w_{el\,12}$	$q_{12} + w_{12} = u_2 - u_1$ Isentropic means adiabatic ($q_{12} = 0$) and reversible ($j_{12} = 0$): $w_{12} = w_{v12\,rev} = u_2 - u_1 = c_v\,(T_2 - T_1)$ $w_{v12\,rev} = \dfrac{p_1\,v_1}{\kappa - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{\kappa-1} - 1\right] = \dfrac{R_i\,T_1}{\kappa - 1}\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right] = \text{etc.}$	$q_{12} + w_{12} = u_2 - u_1 = c_v\,(T_2 - T_1)$ $w_{12} = w_{v\,12} = \dfrac{R_i}{n - 1}\,(T_2 - T_1) = c_v\,\dfrac{\kappa - 1}{n - 1}\,(T_2 - T_1) =$ $= \dfrac{p_1\,v_1}{n - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{n-1} - 1\right] = \text{etc.} \quad (n \neq 1)$ Rev. SC: $w_{12} = w_{v12\,rev}$ $q_{12\,rev} = c_v\,\dfrac{n - \kappa}{n - 1}\,(T_2 - T_1) = \dfrac{n - \kappa}{\kappa - 1}\,\dfrac{p_1\,v_1}{n - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{n-1} - 1\right] = \text{etc.} \quad (n \neq 1)$
1st LT open system $q_{12} + w_{t\,12} = h_2 - h_1$ Rev. pressure change work: $w_{p\,12\,rev} = \int_1^2 v\,\mathrm{d}p$ Technical work: $w_{t\,12} = \int_1^2 v\,\mathrm{d}p + j_{12}$	$q_{12} + w_{t\,12} = h_2 - h_1$ Isentropic means adiabatic ($q_{12} = 0$) and reversible ($j_{12} = 0$): $w_{t\,12} = w_{p\,12\,rev} = \kappa\,w_{v\,12\,rev} = h_2 - h_1 = c_p\ (T_2 - T_1)$ $w_{p\,12\,rev} = \kappa\,\dfrac{p_1\,v_1}{\kappa - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{\kappa-1} - 1\right] = \kappa\,\dfrac{R_i\,T_1}{\kappa - 1}\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right] = \text{etc.}$	$q_{12} + w_{t\,12} = h_2 - h_1 = c_p\,(T_2 - T_1)$ $w_{t\,12} = w_{p\,12} = n\,w_{v\,12} = n\,\dfrac{p_1\,v_1}{n - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{n-1} - 1\right] = \text{etc.} \quad (n \neq 1)$ Rev. SC: $w_{p\,12\,rev} = n\,w_{v\,12\,rev} = \dfrac{n\,R_i}{n - 1}\,(T_2 - T_1) = \text{etc.}$ $q_{12\,rev} = c_v\,\dfrac{n - \kappa}{n - 1}\,(T_2 - T_1) = \dfrac{n - \kappa}{\kappa - 1}\,\dfrac{p_1\,v_1}{n - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{n-1} - 1\right] = \text{etc.} \quad (n \neq 1)$
2nd LT / entropy $\mathrm{d}s = \dfrac{\mathrm{d}q + \mathrm{d}j}{T}$ $\mathrm{d}s = \dfrac{\mathrm{d}u + p\,\mathrm{d}v}{T} = \dfrac{\mathrm{d}h - v\,\mathrm{d}p}{T}$	$\mathrm{d}s = \dfrac{\mathrm{d}q + \mathrm{d}j}{T} = 0 \Rightarrow s_2 - s_1 = 0$	$s_2 - s_1 = c_v\,\dfrac{n - \kappa}{n - 1}\,\ln\dfrac{T_2}{T_1} \quad (\text{for } n \neq 1)$ $s_2 - s_1 = c_v\,\ln\dfrac{T_2}{T_1} + R_i\,\ln\dfrac{v_2}{v_1} = c_p\,\ln\dfrac{T_2}{T_1} - R_i\,\ln\dfrac{p_2}{p_1}$

B. Heat transfer: Mashing

Malt and water are mixed in a mash tank. Hot water flows through a half-pipe coil and heats the mash to $80°C$.

Data:

$m_m = 100\,kg$	$m_W = 400\,kg$	$\dot{m}_{HW} = 0.4\,kg/s$
$\vartheta_m = 58°C$	$\vartheta_W = 65°C$	$\vartheta_{HW,\alpha} = 100°C$
$c_{pm} = 3.57$	$c_{pW} = 4.2$	$[kJ/(kgK)]$
$Pr = 4$	$\nu_{HW} = 2 \cdot 10^{-7}\,m^2/s$	$\lambda_{HW} = 0.6\,W/(mK)$
$\rho_{HW} = 1000\,kg/m^3$	$\lambda_s = 30\,W/(mK)$	$D = 1\,m$
$r_i = 35\,mm$	$\delta = 5\,mm$	$\alpha_i = 2000\,W/(m^2K)$

a)

Calculate the initial mixed temperature ϑ_{mix}, then the heat needed to warm the mixture to $80°C$.

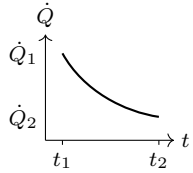
$$\vartheta_{\text{mix}} = \frac{m_m c_{pm} \vartheta_m + m_W c_{pW} \vartheta_W}{m_m c_{pm} + m_W c_{pW}} = 63.77°C$$

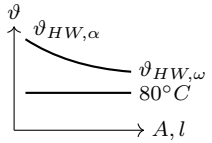
$$Q = \left(m_m c_{pm} + m_W c_{pW}\right)(80 - \vartheta_{\text{mix}}) = 33.05\,MJ$$

b)

Sketch $\dot{Q}$ over heating time. Since the mash gets warmer, the temperature difference to the hot water gets smaller, therefore $\dot{Q}$ decreases.

At t_2, $\vartheta_{HW,\alpha}$ is still $100°C$, but $\vartheta_{HW,\omega}$ is higher than at t_1 because less heat is transferred. The mash temperature is nearly constant over one pass.





c)

At t_2, $\dot{Q} = 5\,kW$. Design the heat-transfer area using the half-pipe Nusselt correlation and treat the half-pipe as a flat plate for k.

$$A = \frac{r_i^2 \pi}{2} = 0.0019\,m^2$$

$$\dot{V} = \frac{\dot{m}}{\rho} = 4 \cdot 10^{-4}\,m^3/s$$

$$v = \frac{\dot{V}}{A} = 0.21\,m/s$$

$$Re = \frac{vL}{\nu} = \frac{v\pi r_i}{\nu} = 115743\,[-]$$

$$Nu = 0.0214\left(Re^{0.8} - 100\right) Pr^{0.4}\left[1 + \left(\frac{L_{\text{char}}}{D}\right)^{2/3}\right] = 510.37$$

$$\alpha = \frac{Nu \cdot \lambda_{HW}}{L_{\text{char}}} = \frac{Nu \cdot \lambda_{HW}}{\pi r_i} = 2785\,W/(m^2K)$$

$$k^{-1} = \frac{1}{\alpha_i} + \frac{\delta}{\lambda_s} + \frac{1}{\alpha}$$

$$k = \frac{1}{k^{-1}} = 975\,W/(m^2K)$$

$$\dot{Q} = \dot{m}_{HW} c_{pW}\left(\vartheta_{HW,\alpha} - \vartheta_{HW,\omega}\right) \Longrightarrow \vartheta_{HW,\omega} = \vartheta_{HW,\alpha} - \frac{\dot{Q}}{\dot{m}_{HW} c_{pW}}$$

$$\vartheta_{HW,\omega} = 100°C - \frac{\dot{Q}}{\dot{m}_{HW} c_{pW}} = 97°C$$

$$\Delta T_m = \frac{\Delta T_a - \Delta T_b}{\ln \Delta T_a - \ln \Delta T_b} = \frac{(100 - 80)\,K - (97 - 80)\,K}{\ln\left(\frac{100-80}{97-80}\right)} = 18.5\,K$$

$$A = \frac{\dot{Q}}{k \Delta T_m} = 0.278\,m^2$$

C. 2nd LT: Exergy of Thermal Energy Storages

Two identical tanks store the same heat amount, either in serial ($T_{S,A} > T_P > T_{S,B}$) or in parallel ($T_{P,A} = T_{P,B}$), with homogeneous tank temperatures.

Data:

Serial mode	$T_{S,A} = 90°C$	$T_{S,B} = 50°C$
Parallel mode	$T_{P,A} = 70°C$	$T_{P,B} = 70°C$
Ambience	$T_{amb} = 20°C = 293\,K$	
Water	$c_{p,W} = 4200\,J/(kgK)$	$= 4.2\,kJ/(kgK)$

a)

Calculate the specific energy content above T_{amb} per kg of water for each tank in both cases.

$$\boxed{u = c_{p,W}\left(T - T_{amb}\right)}$$

$$u_{S,A} = 4.2\,kJ/(kgK) \cdot 70\,K = 294\,kJ/kg$$

$$u_{S,B} = 4.2\,kJ/(kgK) \cdot 30\,K = 126\,kJ/kg$$

$$u_{P,A} = u_{P,B} = 4.2\,kJ/(kgK) \cdot 50\,K = 210\,kJ/kg$$

b)

Calculate the specific entropy difference $s_{S,i} - s_{P,i}$ between serial and parallel operation for both tanks. Water is incompressible, therefore $c_v = c_p$.

$$\boxed{s_{S,i} - s_{P,i} = c_{p,W} \ln\left(\frac{T_{S,i}}{T_{P,i}}\right) + R_i \ln\left(\frac{v_2}{v_1}\right)}$$

$$s_{S,A} - s_{P,A} = 4.2\,kJ/(kgK) \ln\left(\frac{90 + 273}{70 + 273}\right) = 0.24\,kJ/(kgK)$$

$$s_{S,B} - s_{P,B} = 4.2\,kJ/(kgK) \ln\left(\frac{50 + 273}{70 + 273}\right) = -0.25\,kJ/(kgK)$$

c)

Calculate the specific exergy difference $e_{S,i} - e_{P,i}$ between serial and parallel operation for both tanks.

$$\boxed{e_{S,i} - e_{P,i} = h_{S,i} - h_{P,i} - T_{amb}\left(s_{S,i} - s_{P,i}\right)}$$

$$e_{S,A} - e_{P,A} = c_{p,W}\left(T_{S,A} - T_{P,A}\right) - T_{amb}\left(s_{S,A} - s_{P,A}\right) = 13.68\,kJ/kg$$

$$e_{S,B} - e_{P,B} = c_{p,W}\left(T_{S,B} - T_{P,B}\right) - T_{amb}\left(s_{S,B} - s_{P,B}\right) = -10.75\,kJ/kg$$

$$e_S - e_P = \Delta e_A + \Delta e_B = (13.68 - 10.75)\ kJ/kg = 2.93\,kJ/kg$$

D. 2nd LT: Exergy Loss of Heat Transfer

Heat flow through a wall from system A to B for three temperature pairs.

Data:

$\dot{Q} = 100\,kW$	$T_E = T_{amb} = 300\,K$	
i	$T_A = 380\,K$	$T_B = 360\,K$
ii	$T_A = 380\,K$	$T_B = 340\,K$
iii	$T_A = 360\,K$	$T_B = 340\,K$

a)

Formulate the exergy balance of the wall at steady state and derive the exergy loss.

$$\dot{E}_L = \sum_i \dot{E}_{x,i} - \sum_j \left|\dot{E}_{w,j}\right| = \dot{Q}\frac{T_A - T_E}{T_A} - \dot{Q}\frac{T_B - T_E}{T_B}$$

$$\boxed{\dot{E}_L = T_E \dot{S}_{irr} = T_E \frac{T_A - T_B}{T_A T_B}\dot{Q}}$$

b)

Calculate the exergy loss flow $\dot{E}_L$ for all three cases.

$$\dot{E}_{L,i} = 300\,K \cdot \frac{(380 - 360)\,K}{380\,K \cdot 360\,K} \cdot 100\,kW = 4.4\,kW$$

$$\dot{E}_{L,ii} = 300\,K \cdot \frac{(380 - 340)\,K}{380\,K \cdot 340\,K} \cdot 100\,kW = 9.3\,kW$$

$$\dot{E}_{L,iii} = 300\,K \cdot \frac{(360 - 340)\,K}{360\,K \cdot 340\,K} \cdot 100\,kW = 4.9\,kW$$

c)

Calculate the entropy production $\dot{S}_{irr}$ for each case.

$$\dot{E}_L = T_E \dot{S}_{irr} \Longrightarrow \dot{S}_{irr} = \frac{\dot{E}_L}{T_E}$$

$$\dot{S}_{irr,i} = \frac{4400\,W}{300\,K} = 14.7\,W/K$$

$$\dot{S}_{irr,ii} = \frac{9300\,W}{300\,K} = 31\,W/K$$

$$\dot{S}_{irr,iii} = \frac{4900\,W}{300\,K} = 16.3\,W/K$$

E. Stationary flow processes: Compressed-Air Energy Storage

CAES stores electrical energy by compressing air into a cavern; a separate turbine generates power during discharge.

Data:

$$\dot{m}_{air} = 7200\,kg/h = 2\,kg/s \quad T_{2s} = 579\,K$$
$$p_1 = p_5 = 1\,bar \quad p_2 = p_3 = p_4 = 10\,bar$$
$$T_1 = T_3 = 300\,K \quad T_2 = T_4$$
$$n_{comp} = 1.5 \quad n_{exp} = 1.3,\ \kappa = 1.4$$
$$c_{p,air} = 1.004\,kJ/(kgK) \quad R_{air} = 287\,J/(kgK)$$
$$c_{p,stone} = 1.5\,kJ/(kgK)$$

Assumptions: adiabatic compression/expansion with dissipation, polytropic process, ideal thermal storage with $T_2 = T_4$.

a, b)

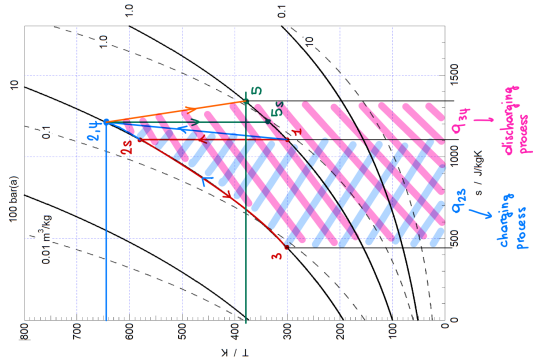
Draw and comment the state points in the T, s-diagram.

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(n_{comp}-1)/n_{comp}} = 300\,K \cdot 10^{0.5/1.5} = 646\,K$$

$$T_2 = T_4$$

$$T_5 = T_4 \left(\frac{p_5}{p_4}\right)^{(n_{exp}-1)/n_{exp}} = 646\,K \cdot 0.1^{0.3/1.3} = 380\,K$$

$1 \to 2$: polytropic compression, $2 \to 3$: heat to storage
$3 \to 4$: heat from storage, $4 \to 5$: polytropic expansion



c)

Calculate the technical work and power of the compressor.

$$w_{t,12} = h_2 - h_1 = c_{p,air}\,(T_2 - T_1) = 347.4\,kJ/kg$$
$$P_{12} = w_{t,12}\dot{m}_{air} = 347.4\,kJ/kg \cdot 2\,kg/s = 694.8\,kW$$

d)

Calculate the technical work and power of the turbine.

$$w_{t,45} = h_5 - h_4 = c_{p,air}\,(T_5 - T_4) = -267.1\,kJ/kg$$
$$P_{45} = |w_{t,45}|\,\dot{m}_{air} = 267.1\,kJ/kg \cdot 2\,kg/s = 534.1\,kW$$

e)

Calculate the isentropic efficiency of the compression.

$$\eta_{s,C} = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{579\,K - 300\,K}{646\,K - 300\,K} = 0.806\,[-]$$

f)

For $4000\,kWh$ of mechanical energy, calculate the required thermal storage energy.

$$P_{34} = \dot{m}_{air} c_{p,air}\,(T_4 - T_3) = 694.8\,kW$$

$$t_{45} = \frac{E_{el}}{P_{45}} = \frac{4000\,kWh}{534.1\,kW} = 7.489\,h$$

$$Q_{th} = P_{34} t_{45} = 694.8\,kW \cdot 7.489\,h = 5203\,kWh = 1.87 \cdot 10^7\,kJ$$

g)

Calculate the required storage mass for stones/gravel.

$$Q_{th} = m c_{p,stone} \Delta T \Longrightarrow m = \frac{Q_{th}}{c_{p,stone}\Delta T} = 36.03\,t$$

h)

Calculate the necessary cave volume at $p = 10\,bar$.

$$pV = mR_{air}T \Longrightarrow V = \frac{\dot{m}_{air} t_{45} R_{air} T_3}{p} = 4642.6\,m^3$$

i)

Calculate the roundtrip efficiency.

$$\eta_{RT} = \frac{|w_{out}|}{w_{in}} = \frac{|w_{t,45}|}{w_{t,12}} = \frac{267.1\,kJ/kg}{347.4\,kJ/kg} = 0.769 = 76.9\%$$

j)

Choose whether to improve the compressor or turbine.

Improve the turbine: it increases the useful output directly, and for the same stored heat and air mass this gives the strongest improvement of delivered electricity and roundtrip efficiency.

F. Gas/Steam Turbines: Micro Steam Turbine

High-pressure superheated steam is expanded from $15\,bar$ to $2\,bar$ in a micro steam turbine instead of a throttling valve. Steam tables are used for the property values; the T, s-graph is attached separately.

Data:

$$p_3 = 15\,bar(a) \quad p_4 = 2\,bar(a)$$
$$\dot{m}_s = 6\,t/h = 1.67\,kg/s \quad \vartheta_3 = 250°C$$
$$\eta_s = 0.8 \quad \eta_m = 0.9,\ \eta_g = 0.96$$
$$P_{el,p} = 20\,kW \quad \eta_p = 0.8,\ c_p = 4.2\,kJ/(kgK)$$
$$T_1 = 120.23°C \quad h_3 = 2923.5\,kJ/kg$$
$$s_3 = 6.7099\,kJ/(kgK) \quad h' = 504.7,\ h'' = 2706.3\,kJ/kg$$
$$s' = 1.5301 \quad s'' = 7.1268 \quad [kJ/(kgK)]$$

a)

Determine the temperature T_2 after the feedwater pump.

$$P_{el,p} = \frac{\dot{m}_s c_p \Delta T}{\eta_p} \Longrightarrow \Delta T = \frac{P_{el,p}\eta_p}{\dot{m}_s c_p}$$

$$T_2 = T_1 + \Delta T = 120.23°C + \frac{20\,kW \cdot 0.8}{1.67\,kg/s \cdot 4.2\,kJ/(kgK)} = 122.5°C$$

b)

Calculate the steam content after the isentropic expansion.

$$x_{4s} = \frac{s_{4s} - s'}{s'' - s'} = \frac{s_3 - s'}{s'' - s'}$$

$$x_{4s} = \frac{6.7099 - 1.5301}{7.1268 - 1.5301} = 0.926 = 92.6\%$$

c)

Calculate the enthalpy h_4 after the polytropic expansion.

$$h_{4s} = (1 - x_{4s})h' + x_{4s}h'' = 2542.3\,kJ/kg$$

$$\eta_s = \frac{h_3 - h_4}{h_3 - h_{4s}} \Longrightarrow h_4 = h_3 - \eta_s\,(h_3 - h_{4s})$$

$$h_4 = 2923.5 - 0.8\,(2923.5 - 2542.3) = 2618.54\,kJ/kg$$

f)

Calculate the specific cycle work of the micro steam turbine.

$$w = w_t - w_p = (h_3 - h_4) - (h_2 - h_1)$$

$$w = (h_3 - h_4) - c_p\,(T_2 - T_1)$$

$$w = (2923.5 - 2618.54) - 4.2\,(122.5 - 120.23) = 295.43\,kJ/kg$$

g)

Calculate the electrical power output of the generator.

$$P_g = \dot{m}_s\,(h_3 - h_4)\,\eta_m \eta_g$$

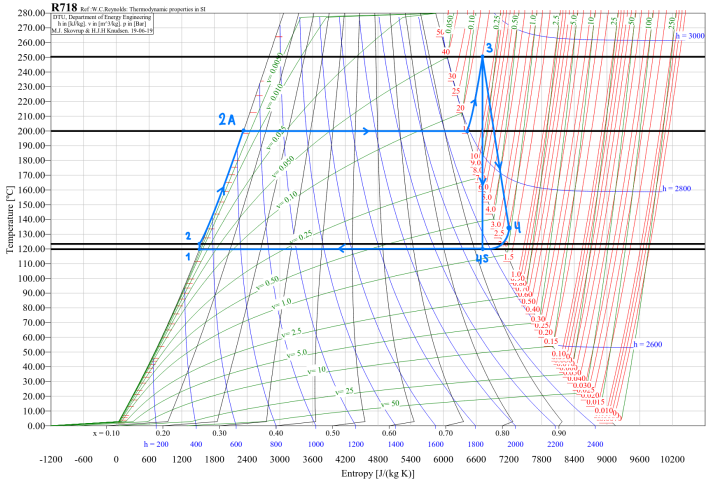
$$P_g = 1.67\,kg/s \cdot (2923.5 - 2618.54)\ kJ/kg \cdot 0.9 \cdot 0.96 = 440\,kW$$

h)

Calculate the thermal efficiency of the micro steam turbine.

$$\eta_{th} = \frac{w}{q_{23}} = \frac{w}{h_3 - h_2} = \frac{295.43\,kJ/kg}{(2923.5 - 4.2 \cdot 120)\,kJ/kg} = 0.122 = 12.2\%$$

d,e)



G. Heat Pumps: Decentralised Cooling System with Waterloop

R290 cooling units reject condenser heat to a closed waterloop. The waterloop releases the heat to ambient air through an air cooler.

Data:

R290 propane	log p,h diagram used separately
$\eta_s = 0.6$	$\eta_{me} = 0.92,\ P_{el} = 1\,kW$
$\Delta T_{SH} = 7\,K$	$\Delta T_{SC} = 5\,K$
$c_{p,W} = 4.2\,kJ/(kgK)$	$\rho_W = 998\,kg/m^3$
$\vartheta_{HW,\alpha} = 35°C$	pump power neglected
$\vartheta_{goods} = 5°C$	$\vartheta_{amb} = 23°C$
$\Delta T_E = 10\,K$	$\Delta T_C = 5\,K,\ \Delta T_{AC} = 3\,K$

a)

Draw the cycle in the log p,h diagram and fill in the state points.

$$\vartheta_E = \vartheta_{goods} - \Delta T_E = 5°C - 10\,K = -5°C \Longrightarrow p_E \simeq 4\,bar$$

$$\vartheta_C = \vartheta_{HW,\alpha} + \Delta T_C = 35°C + 5\,K = 40°C \Longrightarrow p_C \simeq 14\,bar$$

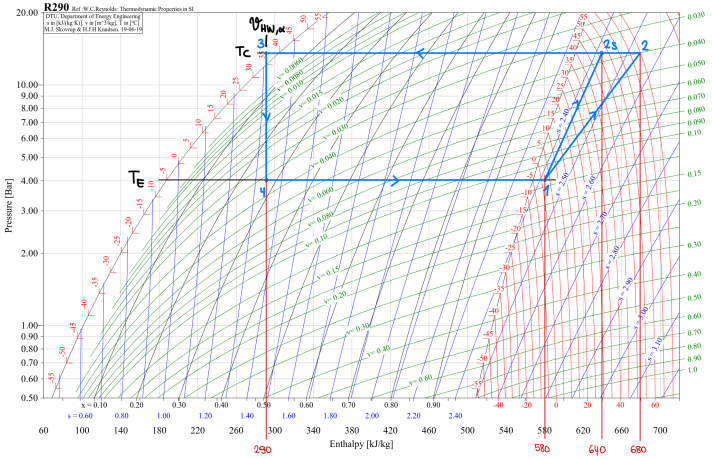
$$1:\ p_E,\ \vartheta_1 = \vartheta_E + \Delta T_{SH} = 2°C \Longrightarrow h_1 \simeq 580\,kJ/kg$$

$$2s:\ p_C,\ s_{2s} = s_1 \Longrightarrow \vartheta_{2s} \simeq 52°C,\ h_{2s} \simeq 640\,kJ/kg$$

$$2:\ \eta_s = \frac{h_{2s} - h_1}{h_2 - h_1} \Longrightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_s} \simeq 680\,kJ/kg$$

$$3:\ p_C,\ \vartheta_3 = \vartheta_C - \Delta T_{SC} = 35°C \Longrightarrow h_3 \simeq 290\,kJ/kg$$

$$4:\ h_4 = h_3,\ p_E,\ \vartheta_4 = \vartheta_E = -5°C$$



b)

Fill the table (by using the log p,h - diagram and calculations).

State point	$\vartheta\,[°C]$	$p\,[bar]$	$h\,[kJ/kg]$
1	2	4	580
2	70	14	680
2s	52	14	640
3	35	14	290
4	-5	4	290

c)

Calculate the cooling capacity $\dot{Q}_E$ of one cooling unit.

$$P_{el}\eta_{me} = \dot{m}\,(h_2 - h_1) \Longrightarrow \dot{m} = \frac{P_{el}\eta_{me}}{h_2 - h_1} = \frac{1\,kW \cdot 0.92}{(680 - 580)\,kJ/kg} = 0.0092\,kg/s$$

$$\dot{Q}_E = \dot{m}\,(h_1 - h_4) = 0.0092\,kg/s \cdot (580 - 290)\,kJ/kg = 2.668\,kW$$

d)

Calculate the cooling-water volume flow for $\vartheta_{amb} = 23°C$.

$$\dot{Q}_{23} = \dot{m}\,(h_2 - h_3) = 0.0092\,kg/s \cdot (680 - 290)\,kJ/kg = 3.588\,kW$$

$$\dot{V}_{CW} = \frac{2\dot{Q}_{23}}{\rho_W c_{p,W}\,(\vartheta_{HW,\alpha} - \vartheta_{amb})} = \frac{2 \cdot 3.588\,kW}{998\,kg/m^3 \cdot 4.2\,kJ/(kgK) \cdot (35 - 23)\,K}$$

$$\dot{V}_{CW} = 14.3 \cdot 10^{-5}\,m^3/s$$

e)

Evaluate the efficiency of one cooling unit.

$$\varepsilon_C = \frac{T_E}{T_C - T_E} = \frac{-5 + 273}{(40 + 273) - (-5 + 273)} = 5.96$$

$$\varepsilon_{CU} = \frac{h_1 - h_4}{h_2 - h_1} = 2.9$$

$$COP_{CU} = \eta_{me}\varepsilon_{CU} = 0.92 \cdot 2.9 = 2.668$$

$$\zeta_{CU} = \frac{COP_{CU}}{\varepsilon_C} = 0.448$$

f)

Heating mode: parameters and expected effect.

The condenser temperature rises to supply building heating, therefore p_C and compressor work increase. The evaporator side is nearly unchanged, so the COP decreases compared with summer operation.