I. Heat transfer

Symbology

Symbol	Quantity	Unit
λ , k	Thermal conductivity	W/(m · K)
$\dot{q}$	Heat flux	W/m^2
$\dot{Q}$	Heat flow	W
α	Heat transfer coefficient	W/(m^2 · K)
a	Thermal diffusivity	m^2/s
ν	Kinematic viscosity	m^2/s

1. Types of heat transfer

1.1 Heat conduction

Occurs with all materials when there is a temperature gradient.

Steady-state conduction (1st Fourier's law):

$$\dot{q} = -\lambda \frac{\mathrm{d}T}{\mathrm{d}x}$$

Transient conduction (2nd Fourier's law 1D):

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial x^2}$$

1.2 Convective heat transfer

This occurs in moving fluids.

Newton's approach:

$$\dot{q} = \alpha \left(T_1 - T_2\right) = \frac{\lambda_{\text{fluid}}}{\delta_{\text{th}}} \left(T_1 - T_2\right)$$

1.3 Radiation

Bodies with a temperature above 0 K emit thermal radiation.

$$\dot{q}_{\text{rad}} = \epsilon \cdot \sigma \cdot \left(T^4 - T_\infty^4\right)$$

where σ is the Stefan-Boltzmann constant:
$\sigma = 5.67 \cdot 10^{-8} \ \left[\frac{W}{m^2 K^4}\right]$

2. Heat conduction and conductivity

2.1 Heat flux

$$\dot{q} = -\lambda \frac{\mathrm{d}T}{\mathrm{d}x} = k \Delta T = \frac{\lambda}{\delta} \Delta T$$

Procedure:

$$\vec{q} \int_0^\delta dx = -\lambda \int_{T_1}^{T_2} dT \Longrightarrow \dot{q} = \frac{\lambda}{\delta} \left(T_1 - T_2\right)$$

2.2 Heat flow

$$\dot{Q} = \frac{\Delta Q}{\Delta t} = \dot{q} A = k A \Delta T = \frac{\lambda}{\delta} A \Delta T$$

2.3 Thermal conduction k^{-1} and Thermal conductivity k

$$k_{\text{tot}}^{-1} = \sum_{i=1}^{n} \frac{\delta_i}{\lambda_i} \implies k_{\text{tot}} = \frac{1}{k_{\text{tot}}^{-1}}$$

3. Heat conduction through a hollow cylinder

$$\dot{Q} = -\lambda A \frac{\mathrm{d}T}{\mathrm{d}r} = -\lambda 2\pi r l \frac{\mathrm{d}T}{\mathrm{d}r}$$

$$\boxed{\dot{Q} = 2\pi l \cdot \frac{T_0 - T_1}{\frac{1}{\lambda} \ln \frac{r_1}{r_0}} = 2\pi r_0 l k \Delta T}$$

With multilayered walls:

$$\dot{Q} = 2\pi l \cdot \frac{T_0 - T_n}{\sum_{i=1}^{n} \frac{1}{\lambda_i} \ln \frac{r_i}{r_{i-1}}} = \boxed{2\pi r_0 l k_0 (T_0 - T_n)}$$

3.1 Thermal conductivity k

$$k_0 = \frac{1}{r_0 \left(\left(\frac{1}{\lambda_1} \ln \frac{r_1}{r_0}\right) + \left(\frac{1}{\lambda_2} \ln \frac{r_2}{r_1}\right) + \cdots + \left(\frac{1}{\lambda_n} \ln \frac{r_n}{r_{n-1}}\right) \right)}$$

$$k_0^{-1} = r_0 \sum_{i=1}^{n} \left(\frac{1}{\lambda_i} \ln \frac{r_i}{r_{i-1}}\right)$$

3.2 Thin-walled pipe approximated as a flat wall

$$\dot{Q}_T = \lambda A \frac{T_0 - T_1}{\delta} \quad ; \quad A = 2\pi l \left(r_0 + \frac{\delta}{2}\right)$$

$$\frac{\dot{Q}_T}{\dot{Q}_W} = \frac{\delta}{\left(r_0 + \frac{\delta}{2}\right) \ln \left(1 + \frac{\delta}{r_0}\right)} = \frac{\delta}{r_0} \frac{1}{\left(1 + \frac{\delta}{2 r_0}\right) \ln \left(1 + \frac{\delta}{r_0}\right)}$$

4. Convective heat transfer

4.1 Reynolds number

It compares inertial to viscous forces.

$$\boxed{Re = \rho\, c \frac{L_{\text{char}}}{\eta} = c \frac{L_{\text{char}}}{\nu}}$$

$Re < 2300$: Laminar flow;
$Re \approx 2300$: Transition zone;
$Re > 2300$: Turbulent flow

4.2 Prandtl number

It compares momentum diffusivity to thermal diffusivity. Relates the relative thickness of the velocity and thermal boundary layers.

$$Pr = \frac{\nu}{a} = \frac{\delta_{\text{fluid}}}{\delta_{\text{th}}} = \frac{\eta c_p}{\lambda} \quad ; \quad a = \frac{\lambda}{\rho c_p} \quad ; \quad \nu = \frac{\eta}{\rho}$$

4.3 Nusselt number

Measures convective heat transfer relative to pure conduction across a boundary layer and the heat transfer coefficient α.

$$Nu = \frac{\alpha \cdot L_{\text{char}}}{\lambda_{\text{fl}}} = \dot{q} \frac{L_{\text{char}}}{\lambda \Delta T_{\text{char}}}$$

$$\alpha = \frac{Nu \cdot \lambda}{L_{\text{char}}}$$

4.3.1 Forced convection

$$\boxed{Nu = c \cdot Re^m \cdot Pr^n}$$

where c is a constant.

For laminar flow over a flat plate:

$$Nu = 0.664 \cdot \sqrt{Re} \cdot \sqrt[3]{Pr}$$

For a flow in the transition zone (Tz):

$$Nu_{\text{Tz}} = \sqrt{Nu_{\text{lam}}^2 + Nu_{\text{turb}}^2}$$

4.4 Grashof number

It compares buoyancy to viscous forces.

$$Gr = g \beta \Delta T \frac{L^3}{\nu^2} \quad ; \quad g = 9.81 \text{ N/kg}$$

$$\beta \Delta T = \frac{\rho_\infty - \rho_0}{\rho_\infty} = \frac{\Delta \rho}{\rho_\infty}$$

$$Gr = g \frac{\rho_\infty - \rho_0}{\rho_\infty} \frac{L^3}{\nu^2}$$

4.4.1 Natural convection

Peclet number

It compares advective to diffusive heat transport, i.e., the strength of convection relative to thermal diffusion.

$$Pe = Re Pr$$

Rayleigh number

It combines buoyancy driving and thermal diffusion, and it measures the intensity of natural convection.

$$Ra = Gr Pr$$

5. Heat exchanger

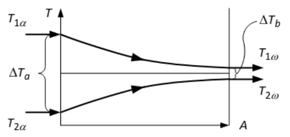
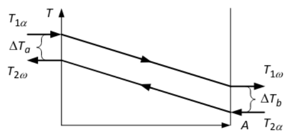
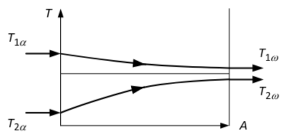
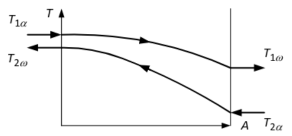
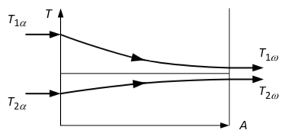
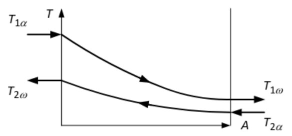
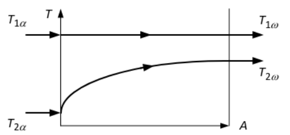
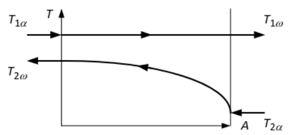
5.1 Energy balance

$$\dot{Q} = \dot{m} c_p \Delta\vartheta = C P \Delta\vartheta$$

$$\dot{Q}_1 = \dot{Q}_2$$

$$\dot{m}_1 c_{p1} \Delta\vartheta_1 = \dot{m}_2 c_{p2} \Delta\vartheta_2 \iff CP_1 \Delta\vartheta_1 = CP_2 \Delta\vartheta_2$$

5.2 Average temperature difference

CP_1, CP_2	Cocurrent	Counter current
$CP_1 = CP_2$ $\Delta T_1 = \Delta T_2$		
$CP_1 > CP_2$ $\Delta T_1 < \Delta T_2$		
$CP_1 < CP_2$ $\Delta T_1 > \Delta T_2$		
$CP_1 = \infty$ $\Delta T_1 = 0$		

5.2.1 Logarithmic mean temperature difference (LMTD)

$$\boxed{\Delta T_m = \frac{\Delta T_a - \Delta T_b}{\ln \dfrac{\Delta T_a}{\Delta T_b}}}$$

$$\Delta T_a = |\vartheta_{1,a} - \vartheta_{2,a}| \quad ; \quad \Delta T_b = |\vartheta_{1,b} - \vartheta_{2,b}|$$

5.2.2 Mean temperature heat flow

$$\boxed{\dot{Q} = kA\Delta T_m}$$

5.2.3 Heat capacity rate ratio (Capacity ratio)

It measures the relative ability of stream 1 vs. stream 2 to carry heat per kelvin:

$$CP_1\,(\vartheta_{1,\alpha} - \vartheta_{1,\omega}) = CP_2\,(\vartheta_{2\omega} - \vartheta_{2,\alpha})$$

$$R_1 = \frac{CP_1}{CP_2} = \frac{\vartheta_{2,\omega} - \vartheta_{2,\alpha}}{\vartheta_{1,\alpha} - \vartheta_{1,\omega}} = \frac{\Delta\vartheta_2}{\Delta\vartheta_1}$$

5.3 Co-current vs countercurrent flow heat exchanger

$$\dot{Q} = kA_{\text{co}}\Delta T_{m,\text{co}} = kA_{\text{count}}\Delta T_{m,\text{count}}$$

$$\frac{A_{\text{co}}}{A_{\text{count}}} = \frac{\Delta T_{m,\text{count}}}{\Delta T_{m,\text{co}}}$$

$$\frac{A_{\text{co}}}{A_{\text{count}}} = \frac{\dot{Q}_{\text{co}}}{\dot{Q}_{\text{count}}} \cdot \frac{\Delta T_{m,\text{count}}}{\Delta T_{m,\text{co}}}, \quad \text{if } \dot{Q}_1 \neq \dot{Q}_2$$

II. The 2nd law of thermodynamics

6. Recap of the 1st law

6.1 Closed systems

$$q_{12} + w_{12} = u_2 - u_1$$

6.2 Open systems

$$q_{12} + w_{12} = h_2 - h_1 + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\,(z_2 - z_1)$$

$$w_{t12} = \int v\,dp + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\,(z_2 - z_1) + j_{12}$$

$$q_{12} + \int v\,dp + j_{12} = h_2 - h_1 \overset{\text{ideal gas}}{=\!=\!=} c_p\,(T_2 - T_1)$$

7. The 2nd law

7.1 Statements

$$\boxed{\text{Energy} = \text{Exergy} + \text{Anergy}}$$

Processes	**Reversible**	**Irreversible**
Exergy	Conserved	Reduced → Loss
Entropy	Constant	Increases → Production

7.2 Entropy

Entropy S: extensive state variable [J/K]

Specific entropy s: intensive state variable [J/kgK]

Entropy flow $\dot{S}$: entropy over time [W/K]

8. Entropy balance equation

$$\boxed{dS = dS_Q + dS_m + dS_{irr}}$$

8.1 Entropy balance terms

- Heat transfer across the system boundary:

$$dS_Q = \frac{dQ}{T}$$

- Mass flow across the system boundary:

$$dS_{\text{m}} = \dot{m}\,s\,dt \quad ; \quad \dot{S}_m = \dot{m}\,s$$

- Irreversible processes inside the system:

$$\dot{S}_{irr}(t) \begin{cases} dS_{irr} > 0 & \text{irreversible processes} \\ dS_{irr} = 0 & \text{reversible processes} \\ dS_{irr} < 0 & \text{impossible} \end{cases}$$

8.2 Entropy balance equation for closed systems

$$dS = dS_Q + \cancelto{0}{dS_m} + dS_{irr}$$

$$dS = \dot{S}_Q\,dt + \dot{S}_{irr}\,dt = \left(\frac{\dot{Q}}{T} + \dot{S}_{irr}\right)dt$$

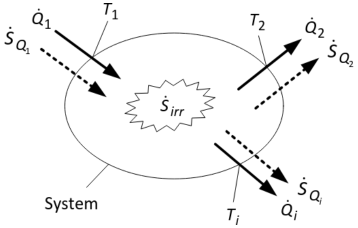
8.2.1 Closed system entropy flow balance equation

$$\frac{dS}{dt} = \sum_i \dot{S}_{Qi} + \dot{S}_{irr} \quad ; \quad \dot{S}_{irr} \geq 0$$

For steady-state case:

$$\dot{S}_{irr} = -\sum_i \dot{S}_{Qi} = -\sum_i \frac{\dot{Q}_i}{T_i} \geq 0$$

8.2.2 Visual representation



$$\boxed{\sum_i \dot{S}_{Qi} = \sum_i \frac{\dot{Q}_i}{T_i} = -\dot{S}_{irr}}$$

8.3 Entropy balance equation for open systems

$$\frac{dS}{dt} = \sum_\alpha \dot{m}_\alpha s_\alpha - \sum_\omega \dot{m}_\omega s_\omega + \sum \dot{S}_Q(t) + \dot{S}_{irr}(t)$$

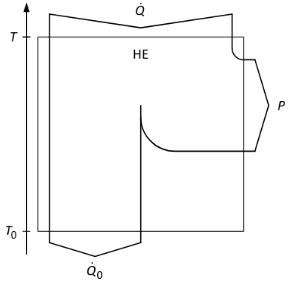
8.3.1 Steady-state flow processes

$$\dot{S}_{irr} = \sum_\omega \dot{m}_\omega s_\omega - \sum_\alpha \dot{m}_\alpha s_\alpha - \sum \dot{S}_Q \geq 0$$

8.3.2 Steady-state and adiabatic flow processes

$$\dot{S}_{irr} = \sum_\omega \dot{m}_\omega s_\omega - \sum_\alpha \dot{m}_\alpha s_\alpha \geq 0$$

9. The thermal engine



$$\dot{Q} + \dot{Q}_0 + P = 0$$

$$\eta_{th} = \frac{-P}{\dot{Q}} = 1 - \frac{\left|\dot{Q}_0\right|}{\dot{Q}}$$

$$\dot{S}_{irr} = -\sum_{i=1}^{n} \dot{S}_{Qi} = -\frac{\dot{Q}}{T} - \frac{\dot{Q}_0}{T_0} \geq 0$$

$$-\dot{Q}_0 = T_0\left(\frac{\dot{Q}}{T} + \dot{S}_{irr}\right)$$

9.1 Carnot efficiency

$$\eta_{th} = 1 - \frac{T_0}{T} - \cancelto{0,\ \text{ideal}}{\frac{T_0\dot{S}_{irr}}{\dot{Q}}}$$

$$\eta_{th,max} = \eta_C(T, T_0) = 1 - \frac{T_0}{T}$$

10. Entropy S as a state variable

$$ds = \frac{dq + dj}{T} = \frac{du + p\,dv}{T} = \frac{dh - v\,dp}{T}$$

$$T\,ds = dq + dj \implies T\,(s_2 - s_1) = q_{12} + j_{12}$$

General form: $\displaystyle\int_1^2 T\,ds = q_{12} + j_{12}$

10.1 Reversible processes

10.1.1 Clausius entropy relation

$$\boxed{\delta q_{\text{rev}} = T\,ds \iff \delta Q_{\text{rev}} = T\,dS}$$

10.1.2 Work W and heat Q

$$w_{p,\,rev} = \int v\,dp$$

$$Q_{rev} = \int T\,dS$$

10.2 Dissipation j as a state variable

10.2.1 Dissipation

$$w_{t12} = w_{p12} + j_{12} \iff w_{p12} = w_{t12} - j_{12}$$

$$dq_{rev} + dw_{v,\,rev} = du$$

$$T\,ds = dq + T\,ds_{irr} = dq_{rev} + dj$$

$$\int_1^2 T\,ds = q_{12} + j_{12}$$

10.2.2 For closed systems

$$ds = \frac{du + p\,dv}{T} = \frac{c_v\,dT}{T} + \frac{R_i\,dv}{v}$$

$$\boxed{s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R_i \ln\frac{v_2}{v_1}}$$

10.2.3 Work change

All the works can be calculated with a temperature change:

$$q_{12} + w_{t12} = h_2 - h_1 + \frac{c_2^2 - c_1^2}{2} + g\,(z_2 - z_1)$$

$$q_{12} + \int v\,dp + j_{12} = h_2 - h_1 = c_p\,(T_2 - T_1)$$

$$\int T\,ds = q_{12} + j_{12} = \boxed{\int T\,ds + \int v\,dp = h_2 - h_1}$$

$$\boxed{T\,ds + v\,dp = h_2 - h_1 \iff T\,ds - p\,dv = u_2 - u_1}$$

$$T\,ds + v\,dp = dh\,T\,ds - p\,dv = du$$

10.3 Reversible state changes with perfect gases

10.3.1 Open system

$$q_{12} + j_{12} + w_{p,rev} = h_2 - h_1$$

$$\int_1^2 T\,ds + \int_1^2 v\,dp = c_p\,(T_2 - T_1)$$

10.3.2 Closed system

$$q_{12} + j_{12} + w_{v,rev} = u_2 - u_1$$

$$\int_1^2 T\,ds - \int_1^2 p\,dv = c_v\,(T_2 - T_1)$$

10.4 Polytropic state changes

$$T\,ds = dq + dj = du + p\,dv = dh - v\,dp$$

$$du + p\,dv = c_v\,dT + p\,dv \quad ; \quad dh - v\,dp = c_p\,dT - v\,dp$$

$$dw_v = c_v \frac{k-1}{n-1}\,dT = -p\,dv$$

For the change in entropy of the polytropic state change:

$$ds = c_v \frac{n-k}{n-1}\frac{dT}{T}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\boxed{s_2 - s_1 = c_v \frac{n-k}{n-1} \ln\frac{T_2}{T_1}} \quad n \neq 1$$

$$\boxed{s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R_i \ln\frac{v_2}{v_1} = c_p \ln\frac{T_2}{T_1} - R_i \ln\frac{p_2}{p_1}}$$

Compressor with dissipation: $n > k$
Turbine with dissipation: $n < k$

11. Exergy and anergy

Exergy: useful part of energy that can be completely converted into work under given ambient conditions.

Anergy: remaining part of energy that cannot be converted into work under given ambient conditions.

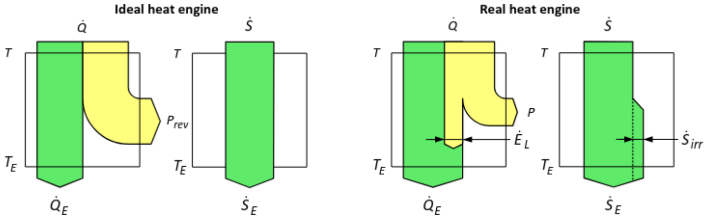
$$\boxed{e = ex + an}$$

11.1 Statements of the 2nd LT

- **I.** In all irreversible processes exergy is converted into anergy
- **II.** Only in reversible processes exergy remains constant
- **III.** It is impossible to convert anergy into exergy

11.2 Exergy

11.2.1 Exergy balance / Exergy loss



$$\dot{E}_L = \sum_\alpha \dot{E}_\alpha - \sum_\omega |\dot{E}_\omega| = \dot{Q}\frac{T_A - T_E}{T_A} + \left(-\dot{Q}\frac{T_B - T_E}{T_B}\right)$$

$$\boxed{\dot{E}_L = T_E \dot{S}_{irr} = T_E \frac{T_A - T_B}{T_A T_B}\dot{Q}}$$

$$\dot{S}_{irr} = \sum_\omega \dot{m}_\omega s_\omega - \sum_\alpha \dot{m}_\alpha s_\alpha - \sum \dot{S}_Q \geq 0, \quad d\dot{S}_Q = \frac{d\dot{Q}}{T}$$

11.2.2 Exergy of heat

$$\dot{E}_Q = P_{rev} = \left(1 - \frac{T_E}{T}\right)\dot{Q} = \eta_C \dot{Q}$$

11.2.3 Exergy of material flows (without mixtures)

$$e = h - h_E - T_E\,(s - s_E) + \frac{c^2}{2} + gz + q\left(1 - \frac{T_E}{T}\right)$$

$$\boxed{e_2 - e_1 = h_2 - h_1 - T_E\,(s_2 - s_1)}$$

Perfect gases

$$e = c_p\left(T - T_E - T_E \ln\frac{T}{T_E}\right) + R_i T_E \ln\frac{p}{p_E}$$

Incompressible liquids (isobaric)

$$e = c_p\left(T - T_E - T_E \ln\frac{T}{T_E}\right)$$

Isobaric for $T < T_E$ (cooling)

$$e = c_p\left(T_E - T - T_E \ln\frac{T_E}{T}\right)$$

III. Steady-state flow processes

12. Adiabatic working cycles

12.1 Compressor

12.1.1 1st law

$$w_{t12} + q_{12} = h_2 - h_1 + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)$$
$$w_{t12} + q_{12} = c_p\left(T_2 - T_1\right) + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)$$
$$P_{12} + \dot{Q}_{12} = \dot{m}\left(w_{t12} + q_{12}\right)$$

12.1.2 Isentropic compression efficiency factor

$$\eta_s = \frac{\text{isentropic work}}{\text{real polytropic work}}$$

$$\eta_s = \frac{\Delta h_s}{\Delta h} = \frac{h_{2,s} - h_1}{h_2 - h_1} = \frac{c_{p,m}\left(T_{2,s} - T_1\right)}{c_{p,m}\left(T_2 - T_1\right)} = \frac{T_{2,s} - T_1}{T_2 - T_1} \leq 1$$

12.2 Turbine

12.2.1 1st law

$$w_{t12} + q_{12} = h_2 - h_1 + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)$$
$$w_{t12} + q_{12} = c_p\left(T_2 - T_1\right) + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)$$
$$P_{12} = \dot{m} w_{t12} \iff -P_{12} = \dot{m}\left|w_{t12}\right|$$

12.2.2 Isentropic turbine efficiency

$$\eta_s = \frac{\text{real polytropic work}}{\text{isentropic work}}$$

$$\eta_s = \frac{-\Delta h}{-\Delta h_s} = \frac{h_1 - h_2}{h_1 - h_{2,s}} = \frac{T_1 - T_2}{T_1 - T_{2,s}}$$

13. Adiabatic flow processes without work

13.1 Throttle / Expansion valve (isenthalpic)

$$h_1 = h_2 \implies j_{12} = -\int_1^2 v\,dp = -R_i T \ln\frac{p_2}{p_1}$$

13.2 Nozzle

$$h_2^0 = h_1^0 \implies h_2 + \frac{1}{2}c_2^2 = h_1 + \frac{1}{2}c_1^2$$
$$c_2 = \sqrt{2\left(h_1 - h_2\right) + c_1^2}$$

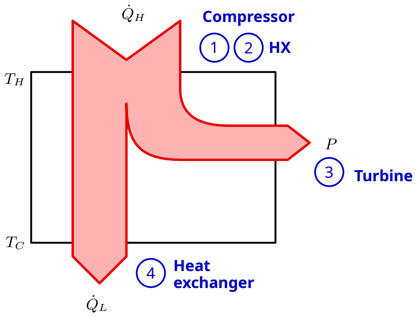
where h^0 is the total enthalpy: $h^0 = h + \dfrac{c^2}{2}$

IV. Thermodynamics cycles

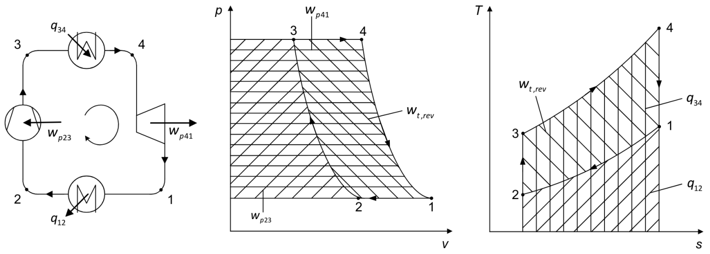
14. Reversible cycle work

Total cycle / sum of all sub-processes:

$$\boxed{w_{t,\,rev} = \int_1^2 v\,dp + \int_2^3 v\,dp + \ldots + \int_n^1 v\,dp = \oint v\,dp = \sum_{i=1}^{n} w_{pi}}$$

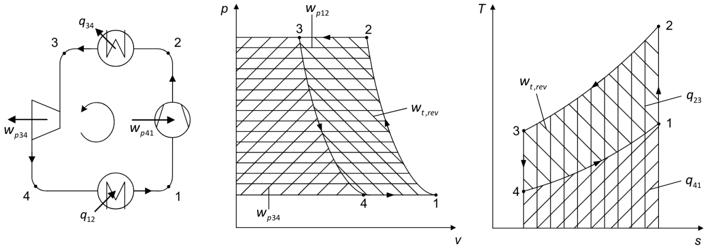


14.1 Clockwise cycle



$$w_{t,rev} = w_{p,23} + w_{p,41} < 0$$
$$\left|w_{t,rev}\right| = \left|q_{34}\right| - \left|q_{12}\right|$$

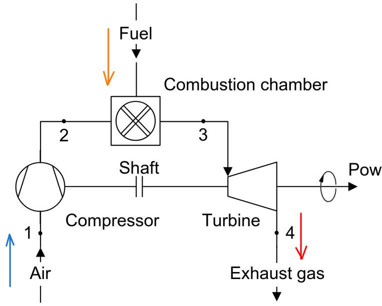
14.2 Counterclockwise cycle

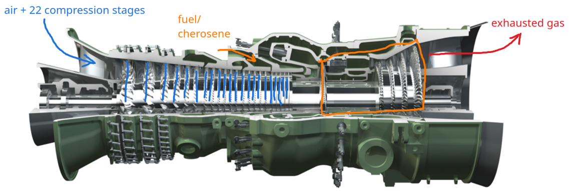


$$w_{t,rev} = w_{p,12} + w_{p,34} < 0$$
$$\left|w_{t,rev}\right| = \left|q_{23}\right| - \left|q_{41}\right|$$

V. Thermal systems with gas turbines

15. Turbine





Power balance:

$$\dot{Q}_{23} + \dot{Q}_{41} + P_C = 0$$

15.1 Joule process

$$\pi = \frac{p_2}{p_1} = \frac{p_3}{p_4}$$

$$-w_C = q_\alpha + q_\omega = q_{23} + q_{41} = c_p\left(T_3 - T_2 + T_1 - T_4\right)$$

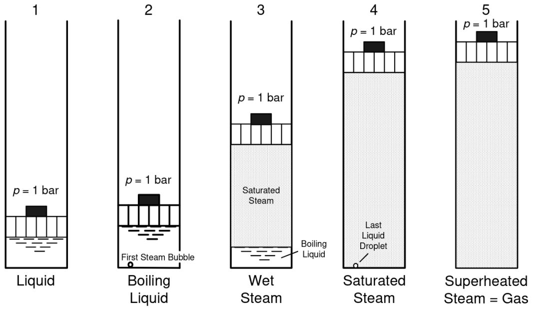
$$\left|w_C\right| = c_p\left(T_3 - T_2\right)\left(1 - \frac{T_4 - T_1}{T_3 - T_2}\right)$$

$$\frac{T_4 - T_1}{T_3 - T_2} = \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}} = \left(\frac{1}{\pi}\right)^{\frac{k-1}{k}}$$

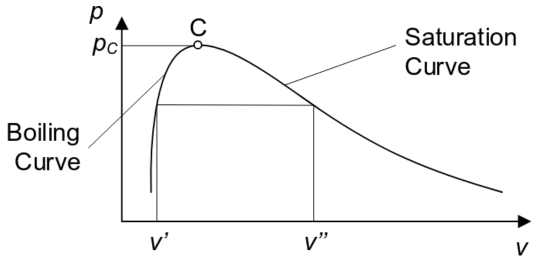
$$\eta_J = \frac{\left|w_C\right|}{q_\alpha} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{1}{\pi}\right)^{\frac{k-1}{k}}$$

VI. Thermal plants with steam turbines

16. Types of steam



17. Boundary curves of the two phase region



17.1 Steam tables

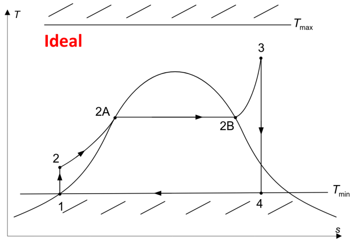
$$v = (1-x)v' + xv'' = v' + x(v''-v')$$

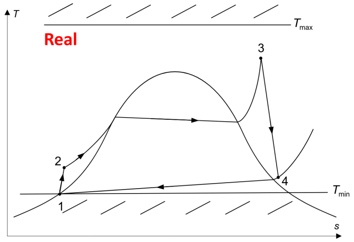
$$h = (1-x)h' + xh'' = h' + x\Delta h_v$$

$$s = (1-x)s' + xs'' = s' + x\left(\frac{\Delta h_v}{T}\right)$$

18. Steam power stations

18.1 Rankine cycle





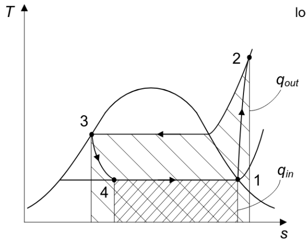
$w_{t12} = h_2 - h_1$ (pump) $|w_{t34}| = h_3 - h_4$ (turbine)

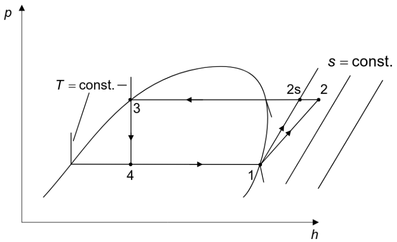
$q_{23} = h_3 - h_2$ (boiler) $|q_{41}| = h_4 - h_1$ (condenser)

$$\eta_{th} = \frac{(h_3-h_4)\overbrace{-(h_2-h_1)}^{\text{pump work}}}{h_3-h_2} \quad ; \quad \eta_s = \frac{h_3-h_4}{h_3-h_{4,s}} \leq 1$$

VII. Heat pumps and Chillers

19. T-s and log p-h diagrams





$w_{t,12} = h_2 - h_1$ (compressor) $h_3 = h_4$ (expansion valve)

$q_{23} = h_2 - h_3$ (condenser) $q_{41} = h_1 - h_4$ (evaporator)

19.1 Isentropic efficiency (compressor)

$$\eta_s = \frac{h_{2,s}-h_1}{h_2-h_1} \leq 1$$

20. Coefficient of performance

20.1 Inner coefficient of performance

20.1.1 Inner COP of a heat pump

$$\varepsilon_{\text{HP}} = \frac{|q_{23}|}{w_{12}} = \frac{\text{heat}}{\text{technical work}} = \frac{h_2-h_3}{h_2-h_1} = \varepsilon_R + 1$$

$$\text{Carnot: } \frac{1}{\eta_C} = \varepsilon_{HP,C} = \frac{T_C}{T_C-T_E} = \frac{T_C}{\Delta T_{\text{lift}}} = \varepsilon_{R,C} + 1$$

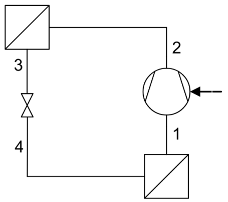
20.1.2 Inner COP of a refrigerator

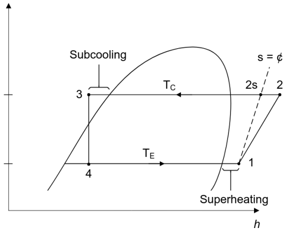
$$\varepsilon_R = \frac{|q_{41}|}{w_{12}} = \frac{h_1-h_4}{h_2-h_1} = \varepsilon_{\text{HP}} - 1$$

$$\text{Carnot: } \varepsilon_{R,C} = \frac{T_E}{T_C-T_E} = \frac{T_E}{\Delta T_{\text{lift}}} = \varepsilon_{HP,C} - 1$$

20.2 Outer coefficient of performance (COP)

20.2.1 COP of a heat pump





$$\text{COP}_{\text{HP}} = \frac{|\dot{Q}_\omega|}{P_{el}} = \frac{|\dot{Q}|}{P_{el}}$$

$$\varepsilon_{HP} > COP_{HP} \quad ; \quad \zeta_{HP} = \frac{COP_{HP}}{\varepsilon_{HP,C}}$$

20.2.2 COP of a refrigerator

$$EER = COP_R = \frac{\dot{Q}_{in}}{P_{el}} = \frac{\dot{Q}_e}{P_{el}}$$

$$\varepsilon_R > EER \quad ; \quad \zeta_R = \frac{EER}{\varepsilon_{R,C}}$$

20.3 Real inner COP

Assumptions: $\zeta = 0.5,\ \Delta T_e = 10K,\ \Delta T_c = 5K$

$$\varepsilon_{HP,C} = \frac{\vartheta_{\text{req}} - \Delta T_c}{(\vartheta_{\text{env}} - \Delta T_c) - (\vartheta_{\text{req}} - \Delta T_e)}$$

$$\varepsilon_{R,C} = \frac{\vartheta_{\text{req}} - \Delta T_e}{(\vartheta_{\text{env}} - \Delta T_C) - (\vartheta_{\text{req}} - \Delta T_e)}$$

$$\varepsilon = \zeta \cdot \varepsilon_C$$

VIII. Data interpolation and extrapolation

21. Interpolation

For any property X, with $X \in \{v, h, s\}$, the general linear interpolation formula is:

$$X = X_1 + \frac{x - x_1}{x_2 - x_1}(X_2 - X_1)$$

where x can be pressure, temperature, or another independent variable.

21.1 Temperature interpolation at constant pressure

If the pressure is already available in the table, but the required temperature lies between two tabulated temperatures ϑ_1 and ϑ_2, then:

$$v = v_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(v_2 - v_1)$$

$$h = h_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(h_2 - h_1)$$

$$s = s_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(s_2 - s_1)$$

21.2 Pressure interpolation at constant temperature

If the temperature is already available in the table, but the required pressure lies between two tabulated pressures p_1 and p_2, then:

$$v = v_{p_1} + \frac{p - p_1}{p_2 - p_1}(v_{p_2} - v_{p_1})$$

$$h = h_{p_1} + \frac{p - p_1}{p_2 - p_1}(h_{p_2} - h_{p_1})$$

$$s = s_{p_1} + \frac{p - p_1}{p_2 - p_1}(s_{p_2} - s_{p_1})$$

21.3 Double interpolation in pressure and temperature

If both pressure and temperature are between table values, first interpolate with respect to temperature at p_1 and p_2, then interpolate with respect to pressure.

Let: $p_1 < p < p_2 \qquad ; \qquad \vartheta_1 < \vartheta < \vartheta_2$

For any property $X \in \{v, h, s\}$:

$$X_{p_1} = X_{11} + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(X_{12} - X_{11})$$

$$X_{p_2} = X_{21} + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(X_{22} - X_{21})$$

$$X = X_{p_1} + \frac{p - p_1}{p_2 - p_1}(X_{p_2} - X_{p_1})$$

21.4 Saturated steam values

For saturated steam properties at the top of the steam table, the same pressure interpolation formula can be used:

$$X'' = X_1'' + \frac{p - p_1}{p_2 - p_1}(X_2'' - X_1'')$$

where:

$$X'' \in \{v'', h'', s''\}$$

22. Extrapolation

For any property X, with $X \in \{v, h, s\}$:

$$X = X_1 + \frac{x - x_1}{x_2 - x_1}(X_2 - X_1)$$

where x is outside the interval $[x_1, x_2]$.

22.1 Pressure extrapolation at constant temperature

If the required pressure is outside the tabulated pressure range, use the two nearest available tabulated pressures. For any property $X \in \{v, h, s\}$:

$$X = X_{p_1} + \frac{p - p_1}{p_2 - p_1}(X_{p_2} - X_{p_1})$$

where p is outside the interval $[p_1, p_2]$.

For the individual steam-table properties:

$$v = v_{p_1} + \frac{p - p_1}{p_2 - p_1}(v_{p_2} - v_{p_1})$$

$$h = h_{p_1} + \frac{p - p_1}{p_2 - p_1}(h_{p_2} - h_{p_1})$$

$$s = s_{p_1} + \frac{p - p_1}{p_2 - p_1}(s_{p_2} - s_{p_1})$$

22.2 Temperature extrapolation at constant pressure

If the required temperature is outside the tabulated temperature range, use the two nearest available temperatures. For any property $X \in \{v, h, s\}$:

$$X = X_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(X_2 - X_1)$$

For the individual properties:

$$v = v_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(v_2 - v_1)$$

$$h = h_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(h_2 - h_1)$$

$$s = s_1 + \frac{\vartheta - \vartheta_1}{\vartheta_2 - \vartheta_1}(s_2 - s_1)$$

22.3 Saturated steam extrapolation

For saturated steam properties outside the tabulated pressure range:

$$X'' = X_1'' + \frac{p - p_1}{p_2 - p_1}(X_2'' - X_1'')$$

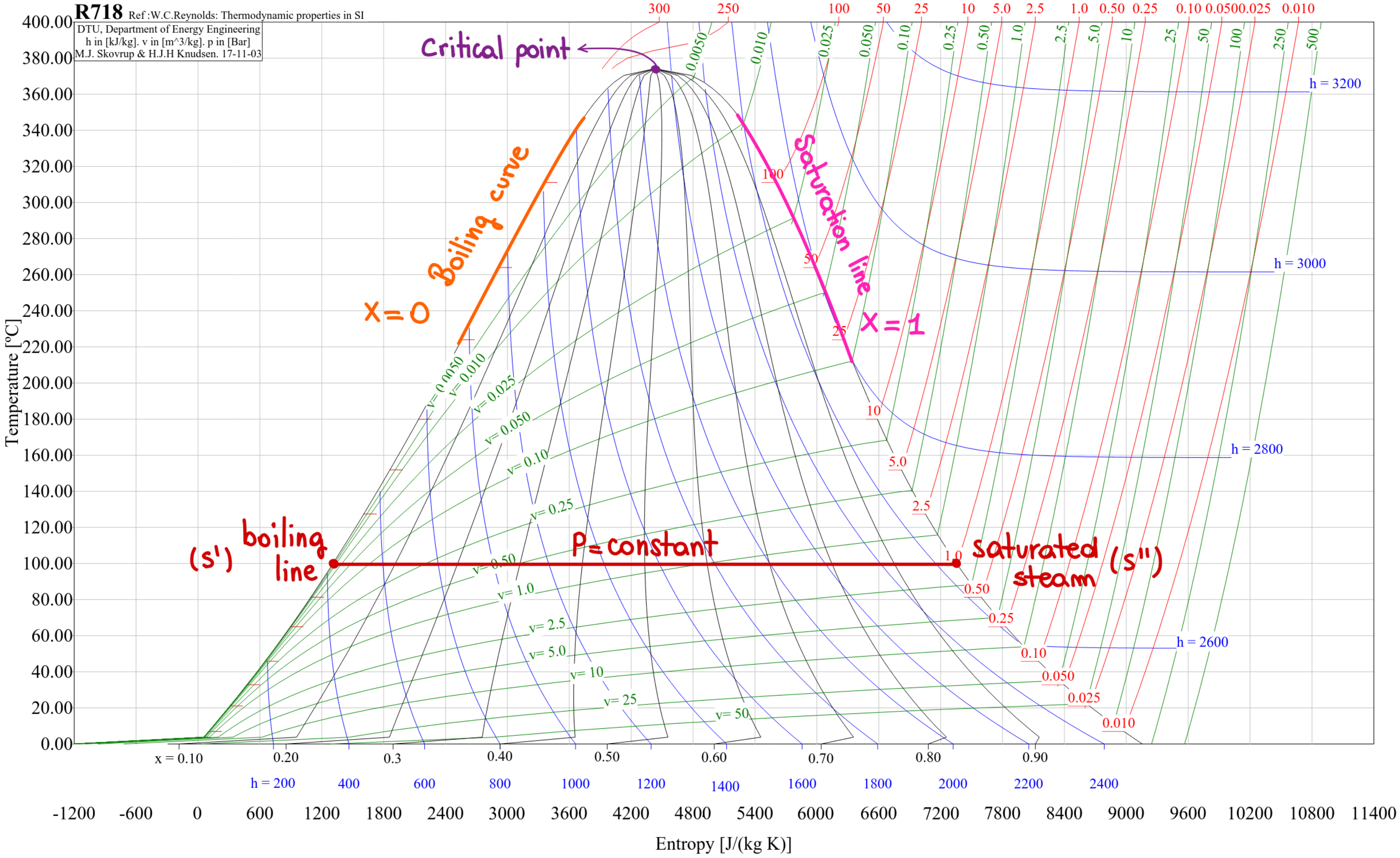
where:

$$X'' \in \{v'', h'', s''\}$$

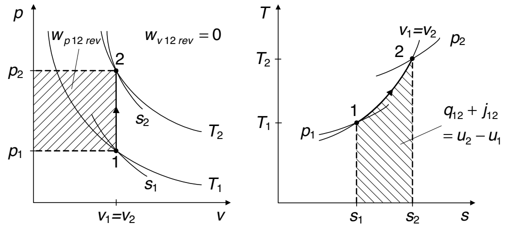
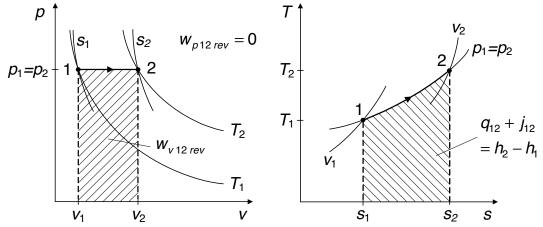
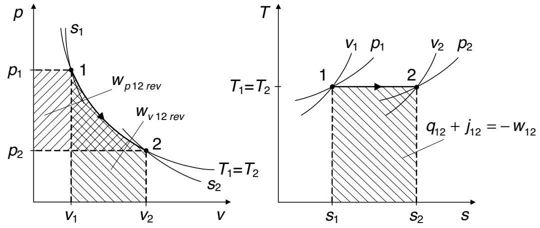
For example, using 25 bar and 50 bar to estimate the value at 100 bar:

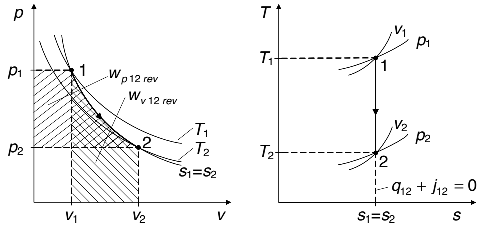
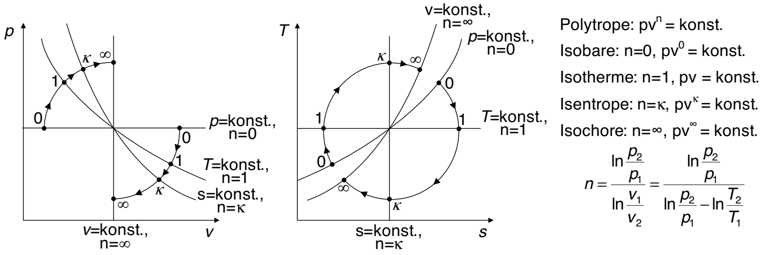
$$X''_{100} = 3X''_{50} - 2X''_{25}$$

A. T-s diagram


R718
Ref :W.C.Reynolds: Thermodynamic properties in SI
DTU, Department of Energy Engineering
h in [kJ/kg]. v in [m^3/kg]. p in [Bar]
M.J. Skovrup & H.J.H Knudsen. 17-11-03
Temperature [ºC]
Entropy [J/(kg K)]
400.00
380.00
360.00
340.00
320.00
300.00
280.00
260.00
240.00
220.00
200.00
180.00
160.00
140.00
120.00
100.00
80.00
60.00
40.00
20.00
0.00
-1200
-600
0
600
1200
1800
2400
3000
3600
4200
4800
5400
6000
6600
7200
7800
8400
9000
9600
10200
10800
11400
x = 0.10
0.20
0.3
0.40
0.50
0.60
0.70
0.80
0.90
h = 200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
h = 2600
h = 2800
h = 3000
h = 3200
300
250
100
50
25
10
5.0
2.5
1.0
0.50
0.25
0.10
0.050
0.025
0.010
v= 0.0050
v= 0.010
v= 0.025
v= 0.050
v= 0.10
v= 0.25
v= 0.50
v= 1.0
v= 2.5
v= 5.0
v= 10
v= 25
v= 50
0.0050
0.010
0.025
0.050
0.10
0.25
0.50
1.0
2.5
5.0
10
25
50
100
250
500
Critical point
Boiling curve
x = 0
Saturation line
x = 1
boiling line
(s')
P = constant
saturated steam
(s'')

Special state changes with perfect gases (without consideration of kinetic and potential energies)

Ideal gas	$v =$ **constant: Isochoric state change**	$p =$ **constant: Isobaric state change**	$T =$ **constant: Isothermal state change**
Therm. state equation $pV = n\,R\,T$ $R = 8.314\,\mathrm{J/mol\,K}$ $pV = m\,R_i\,T$ $R_i = \frac{R}{M_i}$ $p\,v = R_i\,T, \quad \rho = \frac{p}{R_i\,T}$ $R_i = c_p - c_v, \quad \kappa = \frac{c_p}{c_v}$ $c_v = \frac{R_i}{\kappa - 1}, \quad c_p = \frac{\kappa\,R_i}{\kappa - 1}$	 $v = \text{const.}: \quad \frac{p_1}{T_1} = \frac{p_2}{T_2} = \frac{p}{T} = \text{const.}$ $w_{v\,12\,rev} = 0 \quad (\mathrm{d}v = 0)$ $w_{p\,12\,rev} = v\,(p_2 - p_1) = \frac{p_2 - p_1}{\rho}$	 $p = \text{const.}: \quad \frac{v_1}{T_1} = \frac{v_2}{T_2} = \frac{v}{T} = \text{const.}$ $w_{v\,12\,rev} = -p\,(v_2 - v_1) = -R_i\,(T_2 - T_1)$ $w_{p\,12\,rev} = 0 \quad (\mathrm{d}p = 0)$	 $T = \text{const.}: \quad p_1\,v_1 = p_2\,v_2 = p\,v = \text{const.}$ $w_{v\,12\,rev} = -p_1\,v_1\,\ln\frac{v_2}{v_1} = \underbrace{p_1\,v_1\,\ln\frac{p_2}{p_1}}_{[\mathrm{J}]} = \underbrace{R_i\,T\,\ln\frac{p_2}{p_1}}_{[\mathrm{J/kg}]}$ $w_{p\,12\,rev} = w_{v\,12\,rev}$
1st LT closed system $q_{12} + w_{12} = u_2 - u_1$ Rev. volume change work: $w_{v\,12\,rev} = -\int_1^2 p\,\mathrm{d}v$ $w_{v\,12} = w_{v\,12\,rev} + j_{12}$ Overall work: $w_{12} = w_{v\,12} + w_{W\,12} + w_{el\,12}$	$q_{12} + w_{12} = u_2 - u_1$ Rev. SC: $w_{12} = w_{v\,12\,rev} = 0 \quad (\mathrm{d}v = 0)$ $q_{12\,rev} = u_2 - u_1 = c_v\,(T_2 - T_1)$	$q_{12} + w_{12} = u_2 - u_1$ Rev. SC: $w_{12} = w_{v\,12\,rev} = -p\,(v_2 - v_1)$ $q_{12\,rev} = u_2 - u_1 + p\,(v_2 - v_1) = h_2 - h_1$ $q_{12\,rev} = c_p\,(T_2 - T_1)$	$q_{12} + w_{12} = u_2 - u_1 = c_v\,(T_2 - T_1) = 0$ $q_{12} = -w_{12}$ Rev. SC: $q_{12\,rev} = -w_{v\,12\,rev}$ $q_{12\,rev} = p_1\,v_1\,\ln\frac{v_2}{v_1} = -p_1\,v_1\,\ln\frac{p_2}{p_1} = -R_i\,T\,\ln\frac{p_2}{p_1}$
1st LT open system $q_{12} + w_{t\,12} = h_2 - h_1$ Rev. pressure change work: $w_{p\,12\,rev} = \int_1^2 v\,\mathrm{d}p$ Technical work: $w_{t\,12} = \int_1^2 v\,\mathrm{d}p + j_{12}$	$q_{12} + w_{t\,12} = h_2 - h_1$ Rev. SC: $w_{t\,12} = w_{p\,12\,rev} = v\,(p_2 - p_1)$ $q_{12\,rev} = h_2 - h_1 - v\,(p_2 - p_1) = u_2 - u_1$ $q_{12\,rev} = c_v\,(T_2 - T_1)$	$q_{12} + w_{t\,12} = h_2 - h_1$ Rev. SC: $w_{t\,12} = w_{p\,12\,rev} = 0 \quad (\mathrm{d}p = 0)$ $q_{12\,rev} = h_2 - h_1 = c_p\,(T_2 - T_1)$	$q_{12} + w_{t\,12} = h_2 - h_1 = c_p\,(T_2 - T_1) = 0$ $q_{12} = -w_{t\,12}$ Rev. SC: $q_{12\,rev} = -w_{p\,12\,rev}$ $q_{12\,rev} = p_1\,v_1\,\ln\frac{v_2}{v_1} = -p_1\,v_1\,\ln\frac{p_2}{p_1} = -R_i\,T\,\ln\frac{p_2}{p_1}$
2nd LT / entropy $\mathrm{d}s = \frac{\mathrm{d}q + \mathrm{d}j}{T}$ $\mathrm{d}s = \frac{\mathrm{d}u + p\,\mathrm{d}v}{T} = \frac{\mathrm{d}h - v\,\mathrm{d}p}{T}$	$\mathrm{d}s = \frac{\mathrm{d}u}{T} = \frac{c_v\,\mathrm{d}T}{T} \Rightarrow s_2 - s_1 = c_v\,\ln\frac{T_2}{T_1}$	$\mathrm{d}s = \frac{\mathrm{d}h}{T} = \frac{c_p\,\mathrm{d}T}{T} \Rightarrow s_2 - s_1 = c_p\,\ln\frac{T_2}{T_1}$	$\mathrm{d}s = \frac{p\,\mathrm{d}v}{T} = \frac{R_i\,\mathrm{d}v}{v} \Rightarrow s_2 - s_1 = R_i\,\ln\frac{v_2}{v_1}$ $\mathrm{d}s = -\frac{v\,\mathrm{d}p}{T} = -\frac{R_i\,\mathrm{d}p}{p} \Rightarrow s_2 - s_1 = -R_i\,\ln\frac{p_2}{p_1}$

Ideal gas	$s =$ **constant: Isentropic state change**	$n =$ **constant: Polytropic state change**
Therm. state change $pV = n R T$ $R = 8.314\,\mathrm{J/mol\,K}$ $pV = m R_i T$ $R_i = \dfrac{R}{M_i}$ $p v = R_i T, \quad \rho = \dfrac{p}{R_i T}$ $R_i = c_p - c_v, \quad \kappa = \dfrac{c_p}{c_v}$ $c_v = \dfrac{R_i}{\kappa - 1}, \quad c_p = \dfrac{\kappa R_i}{\kappa - 1}$	 $s = \text{const}: \quad p_1 v_1^\kappa = p_2 v_2^\kappa = \text{const.} \quad \Rightarrow \quad \dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} = \left(\dfrac{v_1}{v_2}\right)^{\kappa-1}$ $w_{v\,12\,rev} = \dfrac{p_1 v_1}{\kappa - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{\kappa-1} - 1\right] = \text{etc. (replace)}$ $w_{p\,12\,rev} = \kappa\, w_{v\,12\,rev} = \kappa\, \dfrac{p_1 v_1}{\kappa - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{\kappa-1} - 1\right] = \text{etc. (replace)}$	 $n = \text{const}: \quad p_1 v_1^n = p_2 v_2^n = \text{const.} \quad \Rightarrow \quad \dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\dfrac{v_1}{v_2}\right)^{n-1}$ $w_{v\,12} = \dfrac{p_1 v_1}{n-1}\left[\left(\dfrac{v_1}{v_2}\right)^{n-1} - 1\right] = \text{etc. (replace, } n \neq 1)$ $w_{p\,12} = n\, w_{v\,12} = n\, \dfrac{p_1 v_1}{n-1}\left[\left(\dfrac{v_1}{v_2}\right)^{n-1} - 1\right] = \text{etc. (replace, } n \neq 1)$
1st LT closed system $q_{12} + w_{12} = u_2 - u_1$ Rev. volume change work: $w_{v\,12\,rev} = -\displaystyle\int_1^2 p\,\mathrm{d}v$ $w_{v\,12} = w_{v\,12\,rev} + j_{12}$ Overall work: $w_{12} = w_{v\,12} + w_{W\,12} + w_{el\,12}$	$q_{12} + w_{12} = u_2 - u_1$ Isentropic means adiabatic ($q_{12} = 0$) and reversible ($j_{12} = 0$): $w_{12} = w_{v12\,rev} = u_2 - u_1 = c_v\,(T_2 - T_1)$ $w_{v12\,rev} = \dfrac{p_1 v_1}{\kappa - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{\kappa-1} - 1\right] = \dfrac{R_i T_1}{\kappa - 1}\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right] = \text{etc.}$	$q_{12} + w_{12} = u_2 - u_1 = c_v\,(T_2 - T_1)$ $w_{12} = w_{v\,12} = \dfrac{R_i}{n-1}\,(T_2 - T_1) = c_v\,\dfrac{\kappa - 1}{n - 1}\,(T_2 - T_1) =$ $= \dfrac{p_1 v_1}{n-1}\left[\left(\dfrac{v_1}{v_2}\right)^{n-1} - 1\right] = \text{etc.} \quad (n \neq 1)$ Rev. SC: $w_{12} = w_{v12\,rev}$ $q_{12\,rev} = c_v\,\dfrac{n-\kappa}{n-1}\,(T_2 - T_1) = \dfrac{n-\kappa}{\kappa-1}\,\dfrac{p_1 v_1}{n-1}\left[\left(\dfrac{v_1}{v_2}\right)^{n-1} - 1\right] = \text{etc.} \quad (n \neq 1)$
1st LT open system $q_{12} + w_{t\,12} = h_2 - h_1$ Rev. pressure change work: $w_{p\,12\,rev} = \displaystyle\int_1^2 v\,\mathrm{d}p$ Technical work: $w_{t\,12} = \displaystyle\int_1^2 v\,\mathrm{d}p + j_{12}$	$q_{12} + w_{t\,12} = h_2 - h_1$ Isentropic means adiabatic ($q_{12} = 0$) and reversible ($j_{12} = 0$): $w_{t\,12} = w_{p\,12\,rev} = \kappa\, w_{v\,12\,rev} = h_2 - h_1 = c_p\,(T_2 - T_1)$ $w_{p\,12\,rev} = \kappa\,\dfrac{p_1 v_1}{\kappa - 1}\left[\left(\dfrac{v_1}{v_2}\right)^{\kappa-1} - 1\right] = \kappa\,\dfrac{R_i T_1}{\kappa - 1}\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right] = \text{etc.}$	$q_{12} + w_{t\,12} = h_2 - h_1 = c_p\,(T_2 - T_1)$ $w_{t\,12} = w_{p\,12} = n\, w_{v\,12} = n\,\dfrac{p_1 v_1}{n-1}\left[\left(\dfrac{v_1}{v_2}\right)^{n-1} - 1\right] = \text{etc.} \quad (n \neq 1)$ Rev. SC: $w_{p\,12\,rev} = n\, w_{v\,12\,rev} = \dfrac{n R_i}{n-1}\,(T_2 - T_1) = \text{etc.}$ $q_{12\,rev} = c_v\,\dfrac{n-\kappa}{n-1}\,(T_2 - T_1) = \dfrac{n-\kappa}{\kappa-1}\,\dfrac{p_1 v_1}{n-1}\left[\left(\dfrac{v_1}{v_2}\right)^{n-1} - 1\right] = \text{etc.} \quad (n \neq 1)$
2nd LT / entropy $\mathrm{d}s = \dfrac{\mathrm{d}q + \mathrm{d}j}{T}$ $\mathrm{d}s = \dfrac{\mathrm{d}u + p\,\mathrm{d}v}{T} = \dfrac{\mathrm{d}h - v\,\mathrm{d}p}{T}$	$\mathrm{d}s = \dfrac{\mathrm{d}q + \mathrm{d}j}{T} = 0 \Rightarrow s_2 - s_1 = 0$	$s_2 - s_1 = c_v\,\dfrac{n-\kappa}{n-1}\,\ln\dfrac{T_2}{T_1} \quad (\text{for } n \neq 1)$ $s_2 - s_1 = c_v\,\ln\dfrac{T_2}{T_1} + R_i\,\ln\dfrac{v_2}{v_1} = c_p\,\ln\dfrac{T_2}{T_1} - R_i\,\ln\dfrac{p_2}{p_1}$