Maths refreshing course - Exam 2023
HSLU, Semester 1

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1a)

$x^4 - 24x^2 + 144 = (x^2 - 12)^2$

1b)

$8t^6 + 27b^3 = 2^3 t^{3\cdot 2} + 3^3 \cdot b^3 = (2x^2)^3 + (3y)^3$ (Unsure of next steps)

2a, b)

Unclear

3a)

$(t-5)^2(t+k+k^2)$
$= (t^2 - 10t + 25)(k^2 + k + t)$
$= t^2k^2 + t^2k + t^3 - 10tk^2 - 10tk^2 - 10t^2 + 25k^2 + 25k + 25t$
$= t^3 + t^2k^2 - 10tk^2 + t^2k - 10t^2k - 10t^2 + 25k^2 - 10tk + 25k + 25t$

3b)

$(x+y+2z)^2 = x^2 + xy + 2xz + yx + y^2 + 2yz + 2zx + 4z^2 = x^2 + y^2 + 4z^2 + 2xy + 4xz + 4yz$

4)

$4x^3 - 4x^2 - 11x + 6 = 0$

$$\begin{array}{c|ccc|c} & +4 & -4 & -11 & +6 \\ 2 & & +8 & +8 & -6 \\ \hline & +4 & +4 & -3 & 0 \end{array} \Rightarrow x_1 = 2$$

$$\Downarrow$$

$$4x^2 + 4x - 3 = 0 \to x_{2,3} = \frac{-4 \mp \sqrt{16+48}}{8} = \frac{-4 \mp 8}{8} \Rightarrow x_{2,3} \in \left\{-\frac{3}{2}, \frac{1}{2}\right\}$$

Solution: $x \in \left\{-\frac{3}{2}, \frac{1}{2}, 2\right\}$

5a)

$kx^2 + (k-1)x + \frac{1}{4} = 0 \to \Delta < 0$

$a = k,\ b = k-1,\ c = \frac{1}{4}$

$\Rightarrow (k-1)^2 - 4 \cdot k \cdot \frac{1}{4} = k^2 - 2k + 1 - k = k^2 - 3k + 1$

$\Rightarrow k^2 - 3k + 1 < 0 \Rightarrow k_{1,2} = \frac{3 \mp \sqrt{9-4}}{2}$

$k_1 = \frac{3-\sqrt{5}}{2}; \quad k_2 = \frac{3+\sqrt{5}}{2} \Rightarrow$ The equation has no real solution in the interval $\frac{3-\sqrt{5}}{2} < k < \frac{3+\sqrt{5}}{2}$

5b)

$2x^2 + x + \frac{1}{4} = 0 \to \Delta = \mp\sqrt{1-2} \to \Delta < 0 \to x \in \{\}$

6)

$2x^2 - 2x + 2$

Simmetry axis: $x = \frac{-b}{2a} = \frac{2}{4} = \frac{1}{2}$

Delta (check for intersections with abscissae): $b^2 - 4ac = 4 - 16 = -12 \to \Delta < 0 \to$ No intersection

Vertex: $(V_x, V_y) = \left(\frac{-b}{2a}, f(V_x)\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$

Points:

x	y
0	2
$\frac{3}{2}$	$\frac{7}{2}$

Plot: