Tasks for Test 3 Preparation

A set of optional exercises focusing on Weeks 9 through 12.
(There will be no explanations provided by the teachers for these tasks. Please report any errors found.)

Combinatorics

1. **How many 3-letter words can be formed using the 5 vowels {a, e, i, o, u}?**
   a) With repetition (letters can be reused)
   b) Without repetition (each vowel can only be used once)

2. **How many 4-digit numbers can be created using the digits {1, 2, 3, 4, 5, 6, 7}?**
   a) With repetition (digits can appear multiple times)
   b) Without repetition (each digit can appear only once)
   c) Without repetition and must include at least one even digit
   (Even digits are: {2, 4, 6})

3. **A bakery offers 6 different types of jams, which can be packed into gift boxes. Each gift box has exactly 4 slots for jars.**
   a) **With repetition (the same jam can be used multiple times):**
   How many different types of gift boxes with 4 slots can be created if any jam can be included multiple times?
   b) **Without repetition (each type of jam can only appear once):**
   How many different types of gift boxes with 4 slots can be created if each jam can only appear once in the box?
   c) **With repetition and at least one specific jam (e.g., "strawberry") in the box:**
   How many gift boxes can be created if at least one jar of "strawberry" jam must be included and repetition is allowed?

Conditional Probabilities

4. **In an online dating app, there are two groups of users:**
    - **Group A:** Users with a profile picture. 70% of users in this group receive a message within a week.
    - **Group B:** Users without a profile picture. 30% of users in this group receive a message within a week.
      The app has 60 users in total: 40 with a profile picture and 20 without one.

a) A randomly selected user has received a message within a week. What is the probability that this person has uploaded a profile picture?
b) What is the probability that a person who has uploaded a profile picture receives a message within a week?

Combinatorics and Binomial Distribution

5. **An online shop has introduced a new feature allowing customers to rate their purchases. The probability that a randomly selected customer leaves a positive rating (5 stars) is 60%.**
    - The shop wants to know the likelihood that at least 7 out of 12 randomly selected customers leave a positive rating.
      a) Calculate the probability that at least 7 out of 12 customers leave a positive rating.

Solutions:

1a) 53=1255^3 = 12553=125
1b) 5×4×3=605 \times 4 \times 3 = 605×4×3=60
2a) 74=24017^4 = 240174=2401
2b)
Four out of seven: comb(7,4) = 35,
With orderung: 4! = 24 -> 35*24 = 840 or 7*6*5*4 = 840
2c) Exclusion method (inverse task):
Total combinations with {1, 3, 5, 7} (odd digits): 4! = 24 Total combinations
all possible combinations (4 out of 7): 840
result = total – odd_combindations = 840 – 24 = 816
3a) With repetition: 6^4 = 1296
3b) Without repetition: 6*5*4*3 = 360
3c) With repetition and at least one specific configuration: 1296 – 5^4 = 671
4a) Approximately 82.35%
4b) 70%
5a) 0.6652 or 66.52%