Formula Collection Physics1A, Mechanics

Geometry

Circle: $U = r \cdot 2\pi$, $A = r^2 \cdot \pi$

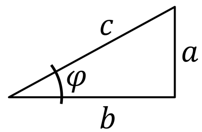
Sphere: $A_{Surface} = r^2 \cdot 4\pi$, $V = r^3 \cdot \frac{4}{3}\pi$

Trigonometry

$\sin\varphi = \frac{a}{c}$

$\cos\varphi = \frac{b}{c}$

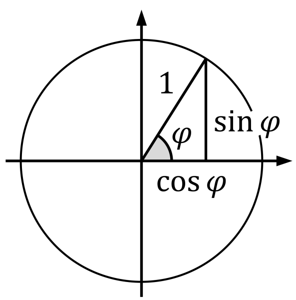
$\tan\varphi = \frac{\sin\varphi}{\cos\varphi} = \frac{a}{b}$

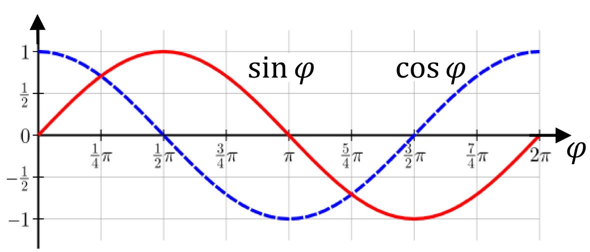


Radians in rad $\leftrightarrow$ Degree in °

$$\varphi[\text{rad}] = \frac{\varphi[°]}{180°} \cdot \pi \leftrightarrow \varphi[°] = \frac{\varphi[\text{rad}]}{\pi} \cdot 180°$$

Trigonometry on the circle





Quadratic equation

$$0 = ax^2 + bx + c, \quad x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

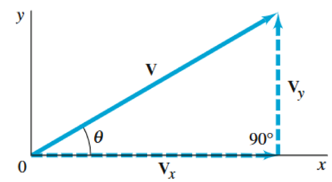
$$\sqrt{b^2-4ac} = D \begin{cases} > 0 \Longrightarrow x_{1/2} = real \\ = 0 \Longrightarrow x_1 = x_2 \\ < 0 \Longrightarrow x_{1/2} = imag \end{cases}$$

Alternative

$$0 = x^2 + px + q, \quad x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

Vectors

$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

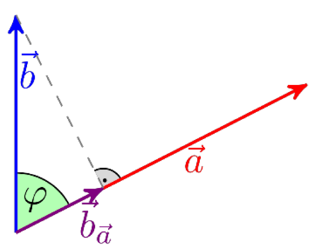


$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}, \qquad \tan\theta = \left(\frac{v_y}{v_x}\right)$$

Scalar product

Projection of

$\vec{b}$ onto $\vec{a}$



$$|\vec{b_{\vec{a}}}| = \vec{a} \cdot \vec{b}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y = |a| \cdot |b| \cos\varphi$$

Quantities in typical letters, they may vary!

Base Units (Mechanics)

Time: t in s

Length: l, s in m

Mass: m in kg

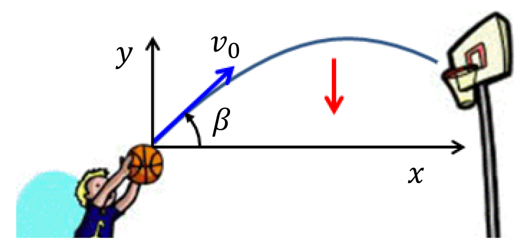
Selected derived units

Energy/Work: E, W in Joule $\text{J} = \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} = \text{Ws}$

Force: F in $\text{N} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$

Power: P in Watt W

Projectile motion



In components:

x-movement	y-movement
$a_x = 0$	$a_y = -g$
$v_x(t) = v_0 \cdot \cos\beta$	$v_y(t) = v_0 \cdot \sin\beta - gt$
$x(t) = (v_0 \cos\beta) \cdot t$	$y(t) = (v_0 \sin\beta) \cdot t - \frac{1}{2}gt^2$
	$v_y^2 = (v_0 \sin\beta)^2 - 2gy$

Parabola

$$y(x) = \tan\beta \cdot x - \frac{g}{2 \cdot (v_0 \cos\beta)^2} x^2$$

Formula Collection Physics1A, Mechanics

Translation (linear motion)		Rotation	
Distance x, arc length s	$x, s = r \cdot \theta, r =$ radius	Angle	θ (Theta)
Velocity $\dot{x}$ Trac/arc speed $\dot{s}$	$\dot{x} = \frac{dx}{dt} = v$ $\dot{s} = r \cdot \dot{\theta} = v_{tan}$	Angular velocity	$\dot{\theta} = \frac{d\theta}{dt} = \omega$ (Omega)
Acceleration	$\ddot{x} = \frac{d^2x}{dt^2} = a, \ddot{s} = r \cdot \ddot{\theta} = a_{tan}$	Angular acceleration	$\ddot{\theta} = \frac{d^2\theta}{dt^2} = \dot{\omega} = \alpha$ (Alpha)
Constant acceleration Average velocity	$a = const.$ $v(t) = \dot{a}(t) = v_0 + a \cdot t$ $x(t) = x_0 + v_0 \cdot t + \frac{1}{2}a \cdot t^2$ $v(t)^2 = v^2 = v_0^2 + 2a(x - x_0)$ $v_{av} = \frac{1}{2}(v + v_0)$	Constant angular acceleration Average angular velocity	$\alpha = konst.$ $\omega(t) = \dot{\theta}(t) = \omega_0 + \alpha \cdot t$ $\theta(t) = \theta_0 + \omega_0 \cdot t + \frac{1}{2}\alpha \cdot t^2$ $\omega(t)^2 = \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ $\omega_{av} = \frac{1}{2}(\omega + \omega_0)$
$a = \dot{v} = \ddot{x} \leftrightarrow x = \int v\, dt, v = \int a\, dt$		$\alpha = \dot{\omega} = \ddot{\theta} \leftrightarrow \theta = \int \omega\, dt, \omega = \int \alpha\, dt$	

Circular motion:

Arc length $s = r \cdot \theta$, (respektive $s = r \cdot \varphi$)

Arc speed $\dot{s} = r \cdot \dot{\theta} = r \cdot \omega = v_{tan}$

Acceleration along arc/path $\ddot{s} = r \cdot \ddot{\theta} = r \cdot \alpha = a_{tan}$

Uniform circular motion:

$\omega = konst.$

Radial (centripetal) acceleration

(CP) $a_{radial} = a_{ZP} = r \cdot \omega^2 = \frac{v^2}{r}$

Angular frequency (Omega) ω ,
frequency f , $\omega = 2\pi \cdot f$

Centripetal force $F_{radial} = F_{CP} = m \cdot a_{CP}$

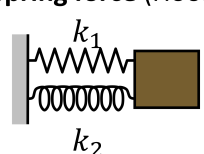
Forces $\vec{F}_{res} = \vec{F}_1 + \vec{F}_2 + \cdots = \sum \vec{F}_k$

Equation of motion $\vec{F}_{res} = m \cdot \vec{a}$

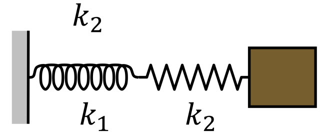
Law of inertia $\vec{F}_{res} = 0 \leftrightarrow \vec{a} = 0 \leftrightarrow \vec{v} = konst$

Law of interaction $\vec{F}_{A\ auf\ B} = -\vec{F}_{B\ auf\ A}$

Spring force (Hooke's model) $\vec{F}_H = -k \cdot \vec{x}$



$k_{tot} = k_1 + k_2$



$k_{tot} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$

Normal force F_N

Frictional forces

Static friction (SF):

$F_{SF} = F_{Pull} \leq \mu_{SF} \cdot F_N$

Kinetic friction (KF):

$F_{KF} = \mu_{KF} \cdot F_N$

Tension

$T = \frac{2m_1m_2}{m_1 + m_2} \cdot g$

Two ropes holding one weight

$T = \frac{mg}{2 \cdot \sin(\theta)}$ or $T_1 = T_2 \frac{\cos(\theta_2)}{\cos(\theta_1)}$

$T_2 = \frac{mg}{\frac{\cos(\theta_2) \cdot \sin(\theta_1)}{\cos(\theta_1)} + \sin(\theta_2)}$

Formula Collection Physics1A, Mechanics

Simple machines *Force • Distance = constant*

Work

$\Delta W = F_x \cdot \Delta x = F\cos\beta \cdot \Delta x = \vec{F}\cdot\Delta\vec{x}, \quad \vec{F} = const.$

Work variable forces:

$\Delta W = \sum_n \vec{F}\cdot\Delta\vec{x}_n = \int_{x_A}^{x_B} \vec{F}\cdot d\vec{x}$

Work = energy transfer

$$W_{Hooke} = \int_{x_A}^{x_B} kx\,dx \rightarrow E_{elast} = \frac{1}{2}k(x_B^2 - x_A^2)$$

$$W_{accel} = F_x\,\Delta x = ma\Delta x \rightarrow E_{kin} = \frac{1}{2}m(v_B^2 - v_A^2)$$

$v = \sqrt{2g\Delta h}$

$W_{pot} = mg\,\Delta s\sin\beta \rightarrow U = E_{pot} = mg\Delta h$

$W_{kin\ friction} = F_{GR}\,\Delta x = \mu_{GR}F_N\Delta x = Q = \text{Wärme}$

Conservation of energy $W_{tot} = 0 \Leftrightarrow E_{tot,A} = E_{tot,B}$

Pulleys

$F_{\text{pull}} = \frac{mg}{n}\ ;\ l_{\text{rope}} = \Delta x \cdot n$

Differential pulley block

$F_{\text{pull}} = mg\left(\frac{R-r}{2R}\right) \qquad W_{\text{lift}} = mg\pi\,(R-r)$

Power (work/time):

$$P = konst = \frac{dW}{dt} = \frac{\vec{F}\,d\vec{s}}{dt} = \vec{F}\cdot\vec{v}$$

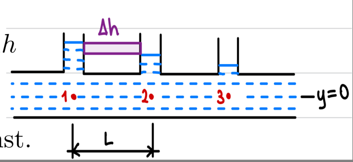
$W = \int P dt$

Average power

$$\langle P\rangle_{\Delta t} = \frac{\int_{t_1}^{t_2} P dt}{t_2 - t_1} = \frac{W}{\Delta t}$$

Piezo tube

$p_1 = p_2 + \Delta p = p_2 + \rho g \Delta h$



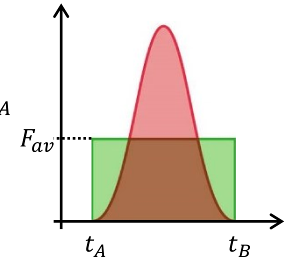
$p + \rho g y + \frac{\rho v^2}{2} + \Delta p = \text{const.}$

Momentum $\vec{p} = m\cdot\vec{v} \quad [N\cdot s]$

Newton generalized: $\vec{F}_{res} = \frac{d\vec{p}}{dt}$

Impulse $\vec{J}$ = Change of momentum $\Delta\vec{p}$

$$\vec{J} = \int_{t_A}^{t_B} \vec{F}\cdot d\vec{x} = \Delta\vec{p} = \vec{p}_B - \vec{p}_A$$



Average force

$$\vec{F}_{av} = \frac{1}{\Delta t}\int_{t_1}^{t_2} \vec{F}\,dt$$

$$= \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1} = \frac{\vec{J}}{\Delta t} = \frac{\vec{p}_B - \vec{p}_A}{t_B - t_A}$$

Cavitation

$$p_1 - p_2 = \frac{I_{\text{vol}}^2}{2A_2^2}\cdot g\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)$$

Momentum conservation

$$\vec{F}_{res} = \sum_k \vec{F}_k = 0$$

$$\Leftrightarrow \vec{p}_{tot} = const$$

$$\Leftrightarrow \vec{p}_{tot,A} = \vec{p}_{tot,B}$$

Completely inelastic collision

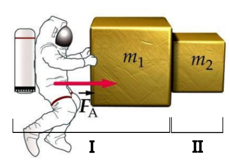
$m_A\vec{v}_{A1} + m_B\vec{v}_{B1}$

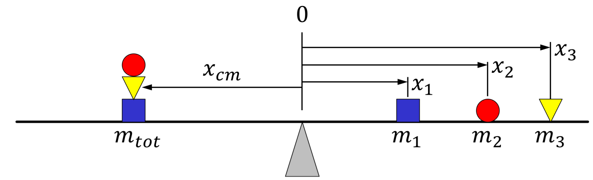
$= m_A\vec{v}_{A2} + m_B\vec{v}_{B2}\ , \qquad \vec{v}_{A2} = \vec{v}_{B2}$

$= (m_A + m_B)\vec{v}_2$

Contact forces

$F_C = m_{\text{ref}}\cdot\frac{F_A}{\sum_n m_n}$





Center of mass coordinates:

(Comparison Balance Beam Scale)

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \sum_i \frac{m_ix_i}{m_{tot}}$$

$$y_{cm} = \frac{m_1y_1 + \cdots}{m_1 + m_2 + \cdots} = \sum_i \frac{m_iy_i}{m_{tot}} = \frac{\int_0^{m_{tot}} y dm}{m_{tot}}$$

Movement of center of mass:

$$m\vec{v}_{cm} = \vec{p} = \vec{p}_1 + \vec{p}_2 + \cdots = \mathrm{m}_1\,\vec{v_1} + \mathrm{m}_2\,\vec{v_2} + \cdots$$

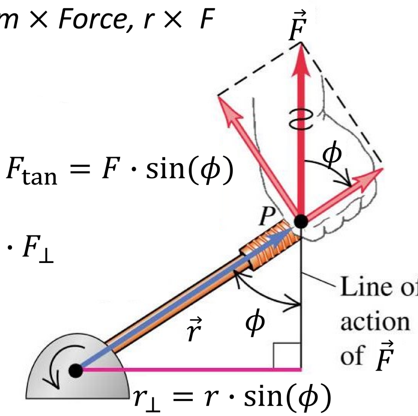
$$m\vec{a}_{cm} = \frac{d\vec{p}}{dt} = m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots = \vec{F}_{res} = \sum \vec{F}_{ext,k}$$

Torque *Lever arm × Force, r × F*

$\vec{M} = \vec{r}\times\vec{F}$

$F_\perp = F_{\tan} = F\cdot\sin(\phi)$

$|\vec{M}| = r_\perp \cdot F = r\cdot F_\perp$



Formula Collection Physics1A, Fluids

Pressure

$$p = \frac{F_\perp}{A} = \frac{dF_\perp}{dA}$$

$$[p] = \frac{\mathrm{N}}{\mathrm{m}^2} = \frac{\mathrm{kg}}{\mathrm{m}\cdot\mathrm{s}^2} = \text{Pascal} = \text{Pa}$$

1 atm = 1.013 bar = 1.013×10^5 Pa

1 bar = 10^5 Pa = 10^3 hPa

Density

$\rho = \frac{m}{V}$ (rho), V: Volume

Fluid pressure, hydrostatic pressure

$$p = p_0 + \rho \cdot g \cdot h$$

p_0: external pressure

h: height of column, depth

Principle of Pascal

$$\frac{F_1}{A_1} = p_1 = p_2 = \frac{F_2}{A_2}$$

Force and torque on sidewall

$$F_{res} = \int_0^h p(x)\mathrm{d}A, \quad p(x) \quad : \quad \text{Druck in Tiefe } x$$

$$x_{CP}\, F_{res} = M_0 = \int_0^h x \cdot p(x)\,\mathrm{d}A$$

Principle of Archimedes

$$F_A = \rho_{Fl} \cdot g \cdot V_{Kö}$$

ρ_{Fl}: density of the liquid

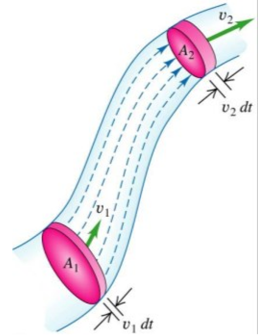
$V_{Kö}$: immersed body volume

Buoyancy

$$V_{Ko} = \frac{|m_{Fl,1} - m_{Fl,2}|}{\rho_{Fl}}$$

CONSERVATION EQUATIONS

Continuity equation



Volume flow

$$I_{Vol} = A_1 \cdot v_1 = A_2 \cdot v_2 = const.$$

v_i: Flow velocity at point i

A_1: Cross-section at point i

Venturi effect

$$p_1 - p_2 = \frac{1}{2} I_{\mathrm{vol}}^2 \cdot g \left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$

Bernoulli's equation along flow line

$$p_1 + \frac{1}{2}\rho\, v_1^2 + \rho g\, h_1 = p_2 + \frac{1}{2}\rho\, v_2^2 + \rho g\, h_2 = konst.$$

Velocities:

Pipe flow

$$v = v_1 = \sqrt{2g(H-h)}$$

Pitot Tube

$$v_0 = \sqrt{\frac{2\rho_W g h}{\rho}} \qquad p_3 = p_1 + \tfrac{1}{2} g v_1^2$$

Viscosity $\boldsymbol{\eta}$ (*eta*)

Shear stress τ (*tau*)

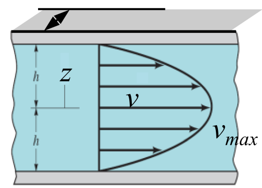
$$\tau = \frac{F_\parallel}{A} = \eta \frac{dv_\parallel}{dz}$$

$F_\parallel$: Shear force

$v_\parallel$: Velocity

$A \parallel v \parallel F, z \perp v \qquad F \sim \frac{A \cdot v}{z}$

Flow profile between two parallel plates



$$v_{\max} = \frac{H^2}{8\eta}\frac{\Delta p}{L}$$

$$v(z) = v_{max} \cdot \left[1 - \left(\frac{z}{h}\right)^2\right] = \frac{3}{2} v_{av} \cdot \left[1 - \left(\frac{z}{h}\right)^2\right]$$

Flow profile in a circular pipe

$$v(r) = \frac{\Delta p \cdot R^2}{4 \cdot L \cdot \eta} \cdot \left[1 - \left(\frac{r}{R}\right)^2\right] = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Δp: loss of pressure — L: length of pipe

R: inner pipe radius — v_{max}: maximum velocity

Hagen-Poiseuille equation

$$I_{Vol} = \pi R^2 \frac{v_{max}}{2} = \frac{\pi R^4}{8\eta}\frac{\Delta p}{L}$$

$$\Delta p = \frac{128 L\eta \cdot I_{\mathrm{vol}}}{\pi \cdot D^4} \qquad \Delta p = \frac{8 L\eta \cdot I_{\mathrm{vol}}}{\pi R^4}$$

L: Pipe length, R: Pipe radius

I_{vol} : volumetric flow rate $\dot{V}$

Pipe outlet with viscous loss

$$v = \sqrt{\left(\frac{8\eta L}{\rho R^2}\right)^2 + 2gh} - \frac{8\eta L}{\rho R^2}$$

Resistance forces

Flow around body:

$$F_{Drag} = \frac{1}{2}\rho_{Fl} v^2 A c_w$$

ρ_{Fl}: density of fluid

v: Flow velocity

A: Cross-sectional area

Flow resistance of a sphere (v small)

$$F_{Stokes} = 6\pi\eta R v$$

R: radius sphere

Looping

$h_{\min} = \frac{5}{2} r \quad | \quad h_{\min} = \left(\frac{5}{2} r + 4\mu_k\right) r$

$mgh = mg(2r) + \frac{1}{2} m v_{\mathrm{top}}^2 \left(+ \int_{\mathrm{track}} F_F ds\right)$

Velocity

$v(\theta)^2 = 2g\,(h - r\,(1 + \sin\theta))$

Tangent force

$F_{\tan}(\theta) = mg\cos\theta$

Resulting force

$F_{\mathrm{net}}(\theta) = m\frac{v^2(\theta)}{r}$