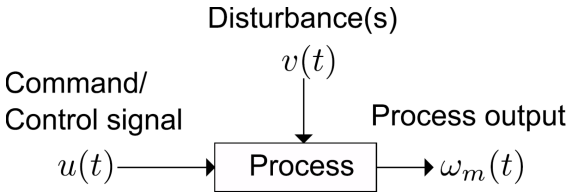
I. Control Theroy and Automation

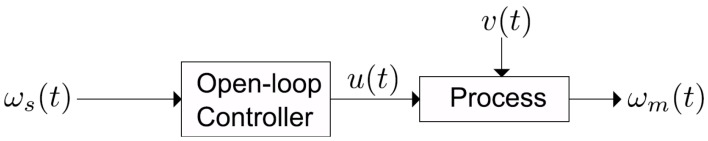
A control system regulates a process using sensors, controllers, actuators, and feedback.

Control theory designs controllers to regulate and optimize dynamic systems.

Uncontrolled system



Open-loop controlled system



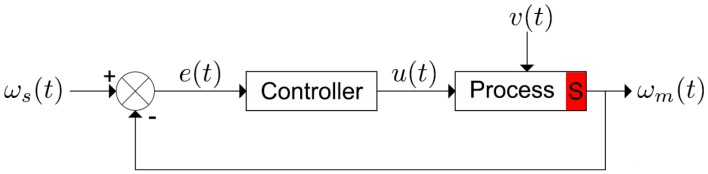
Advantages

- Simple structure: no sensors or controllers needed
- Low cost: fewer components and less complexity

Disadvantages

- No adaptation: cannot correct changes or disturbances
- Unstable performance: quality may decrease over time

Closed-loop controlled system



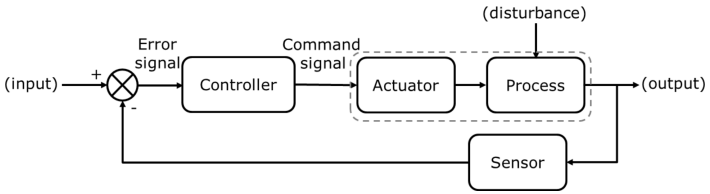
Advantages

- High accuracy: continuous correction improves precision, performance, and efficiency
- Adaptation: reacts to load changes, disturbances, or operating conditions

Disadvantages

- Complex structure: sensors, controllers, and feedback are required
- Higher cost: more components increase total cost
- More maintenance: components need calibration and maintenance

Extended block diagram



Components

Controller

Input: error $e(t)$
Output: command signal $u(t)$

Actuator

Input: command signal $u(t)$
Output: process input signal

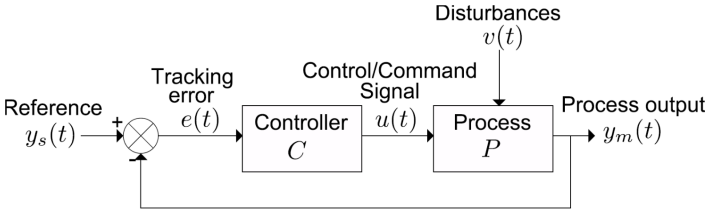
Process

Input: process input signal
Unwanted output: disturbance/noise $v(t)$
Output: actual process output $y_m(t)$

Sensor

Input: actual process output $y_m(t)$
Output: signal to be compared to the desired output signal

Feedback loop in a closed-loop control



$$e(t) = y_s(t) - y_m(t)$$
$$u(t) = C(e(t))$$
$$y_m(t) = P(u(t), v(t))$$

$y_s(t), e(t), u(t), y_m(t)$: Signals
P, C : Systems

Signals vs System

Signals

- Carry information through variations in a physical quantity
- Described as functions of one or more variables
- Can be natural or artificial and are used in communication, measurement, and control

System

- Interconnection of components used to perform a specific function
- Takes input signals, processes or transforms them, and produces output signals
- Describes the relationship between inputs and outputs

Automation pyramid

Level 0. Field/process level
Level 1. Control devices level
Level 2. Supervisory control level
Level 3. Manufacturing planning and execution system (MES)
Level 4. Management/Business level or ERP
Level 5. Digital transformation and AI-driven intelligent ecosystems (cloud level)

Control engineering

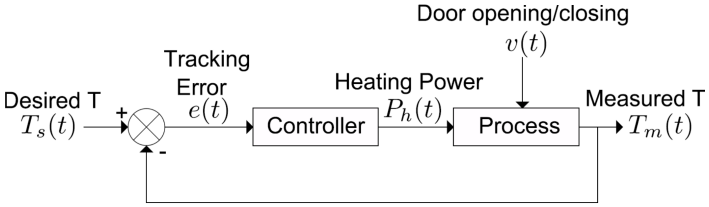
Levels 0-1: real-time control of physical processes using sensors, measurement systems, actuators, and controllers

Automation

Levels 2-5: integration, supervision, optimization, planning, and management of production and business processes

II. Good controllers

Sauna example, closed-loop control



Disturbance rejection

- Ability to reduce the effect of internal or external disturbances on the system output
- Keeps the output close to the desired reference

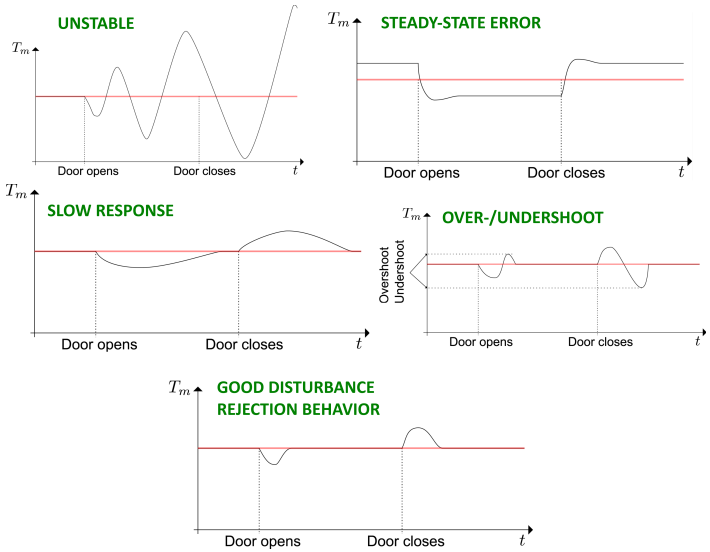
Characteristics

- Disturbance types: internal component changes or external environmental changes
- Effectiveness: controller detects and compensates disturbances
- Control strategies: feedback control reacts after disturbance; feedforward control acts before it affects the output

Performance metrics

- Stability: system remains robust under disturbances
- Steady-state error: final deviation from the reference is minimized
- Overshoot/undershoot: excessive output deviations are reduced
- Rise/settling time: response is fast and stable

Graphical representation



Reference tracking

- Ability to follow the desired reference signal accurately
- Tracking error: difference between reference and output

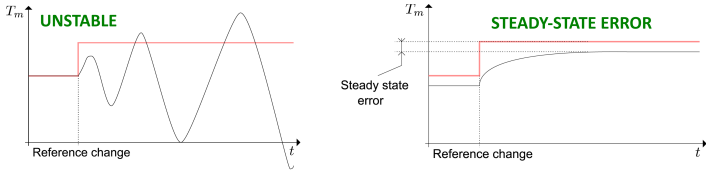
Characteristics

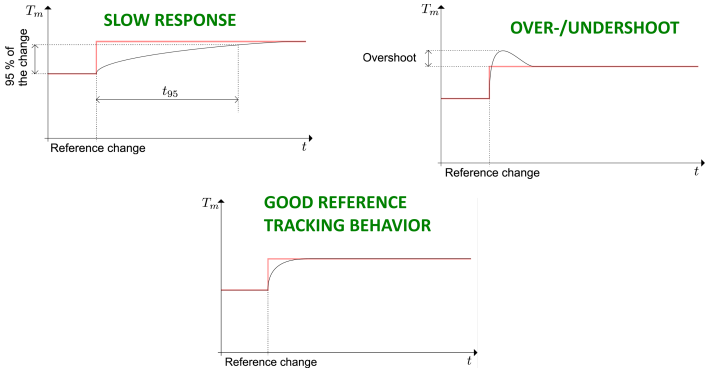
- Tracking accuracy: output follows the reference with minimal deviation
- Effectiveness: controller reduces tracking error under different conditions
- Control strategies: feedback corrects deviations; feedforward anticipates reference changes

Performance metrics

- Stability: tracking remains consistent and robust
- Steady-state error: long-term deviation from the reference is minimized
- Overshoot/undershoot: excessive deviations during tracking are reduced
- Rise/settling time: output reaches the reference quickly and stably

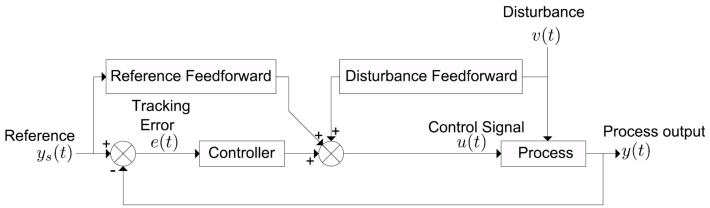
Graphical representation





Feedforward control

- Predictive control strategy that compensates known influences before they affect the output
- Improves performance by anticipating changes instead of only reacting to errors



Reference feedforward

- Compensates reference changes before they affect the system
- Improves tracking by adjusting the control input in advance

Disturbance feedforward

- Compensates measurable disturbances before they affect the output
- Uses a precomputed correction based on the expected disturbance effect

III. Process model development

Damped mass-spring example

Differential equation

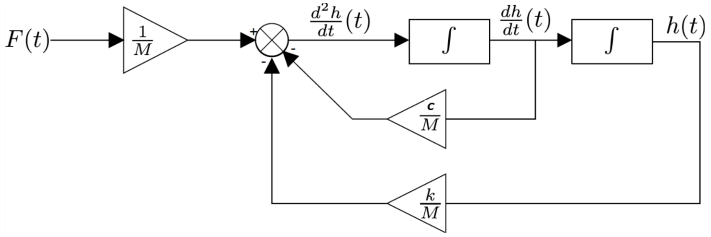
Step 1: Set the differential equation

$$M\frac{d^2h}{dt^2} + c\frac{dh}{dt} + kh = F$$

Step 2: Isolate the term with the highest differential order

$$\frac{d^2h}{dt^2} = \frac{F}{M} - \frac{kh}{M} - \frac{c}{M}\frac{dh}{dt}$$

ODE diagram



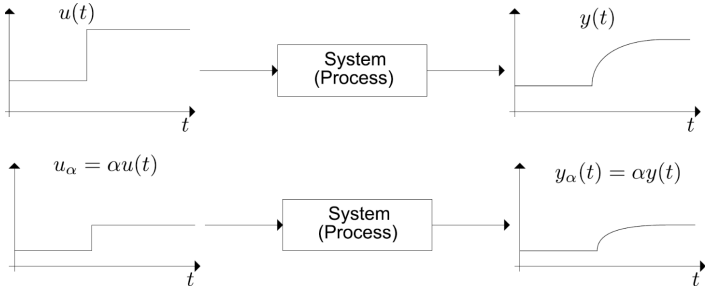
IV. Signals and Systems

Homogeneity (scaling principle)

Scaling the input by a constant also scales the output by the same constant:

- If $u(t) \xrightarrow{h} y(t)$, then $\alpha u(t) \xrightarrow{h} \alpha y(t)$
- If $y(t) = h(u(t))$, then $h(\alpha u(t))$, that is, $h(\alpha u(t)) = \alpha h(u(t))$

Graphical visualization



Homogeneity proof examples

Homogeneous system

$$u(t) \xrightarrow{h} y(t) = 4u(t)$$
$$h(\alpha u(t)) = 4(\alpha u(t)) = \alpha(4u(t)) = \alpha h(u(t))$$
$$\alpha u(t) \xrightarrow{h} \alpha y(t) \qquad \square$$

Non-homogeneous system

$$u(t) \xrightarrow{h} y(t) = u^3(t)$$
$$h(\alpha u(t)) = (\alpha u(t))^3 = \alpha^3 y(t) \neq \alpha y(t)$$
$$h(\alpha u(t)) \neq \alpha h(u(t)) \qquad \square$$

Additivity (superposition principle)

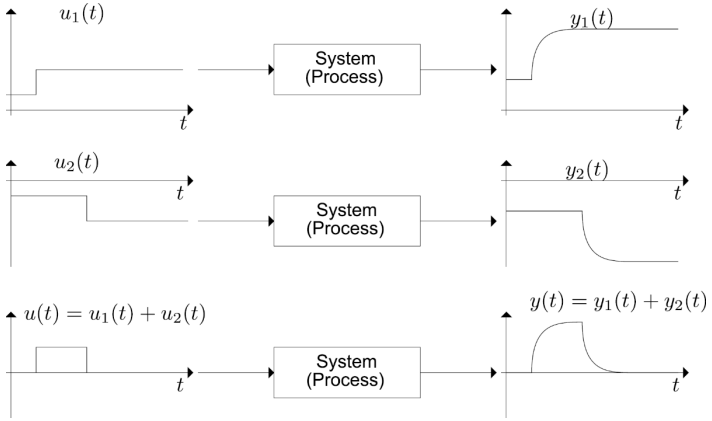
The output of summed inputs equals the sum of the individual outputs:

- If $u_1(t) \to y_1(t)$ and $u_2(t) \to y_2(t)$, then
$$u_1(t) + u_2(t) \to y_1(t) + y_2(t)$$
- If $y_1(t) = h(u_1(t))$ and $y_2(t) = h(u_2(t))$, then
$$h(u_1(t) + u_2(t)) = y_1(t) + y_2(t)$$

That is

$$h(u_1(t) + u_2(t)) = h(u_1(t)) + h(u_2(t))$$

Graphical visualization



Additivity proof examples

Additive system

$$u_1(t) \xrightarrow{h} y_1(t) = 2u_1(t)$$
$$u_2(t) \xrightarrow{h} y_2(t) = 2u_2(t)$$
$$h\big(u_1(t) + u_2(t)\big) = 2\big(u_1(t) + u_2(t)\big)$$
$$= 2u_1(t) + 2u_2(t) = y_1(t) + y_2(t)$$
$$u_1(t) + u_2(t) \xrightarrow{h} y_1(t) + y_2(t)$$ □

Non-additive system

$$u(t) \xrightarrow{h} y(t) = \sin(u(t))$$
$$h(u_+(t) + u_2(t)) = \sin(u_1(t) + u_2(t)) \neq \sin(u_1(t)) + \sin(u_2(t))$$ □

Linear system

A system is linear if it satisfies both the homogeneity (scaling) and additivity principles.

Time-invariant system

System behavior does not change over time. A time-shifted input produces the same time-shifted output:

- If $u(t) \to y(t)$, then
$$u(t - t_0) \to y(t - t_0)$$
- If $y(t) = h(u(t))$, then
$$h(z(t - t_0)) = y(t - t_0)$$

Time-invariance proof example

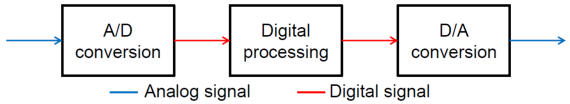
Time-invariant system

$$u(t) \xrightarrow{h} y(t) = u(t) + 7$$
$$h\big(u(t - t_0)\big) = u(t - t_0) + 7 = y(t - t_0)$$
$$u(t - t_0) \xrightarrow{h} y(t - t_0)$$ □

Time-variant system

$$u(t) \xrightarrow{h} y(t) = u(t) + t$$
$$h\big(u(t - t_0)\big) = u(t - t_0) + t$$
$$y(t - t_0) = u(t - t_0) + (t - t_0)$$
$$h\big(u(t - t_0)\big) \neq y(t - t_0)$$ □

Digitalization



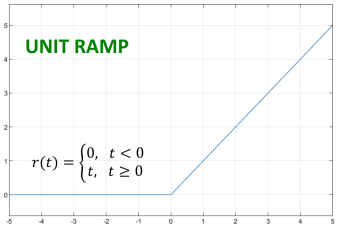
Time discretization

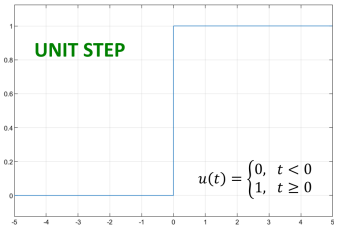
- Sampling: continuous signal measured at fixed time intervals
- Sampling rate: number of samples per second [Hz or kHz]

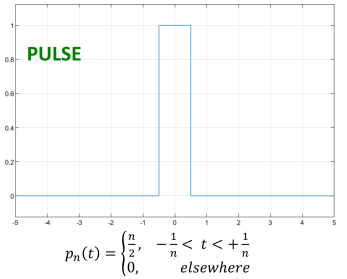
Amplitude discretization

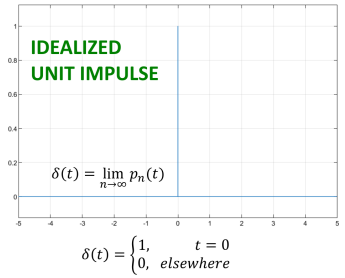
- Quantization: signal values are rounded to discrete levels
- Quantization type: uniform uses equal steps; non-uniform uses variable steps
- Quantization step: coarse steps give lower precision; fine steps give higher precision

Basic input signals









V. Linearization

Steps for linearizing a function

Step 1. Determine the nonlinear differential equation:
$$M\ddot{h} + c\dot{h} = Mg - kh^3$$

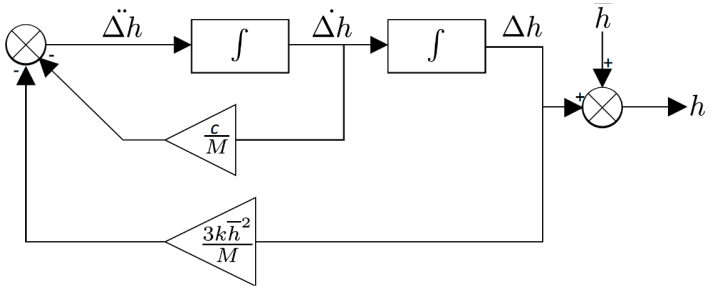
Step 2. Define $f(\ddot{h}, \dot{h}, h) = 0$:
$$M\ddot{h} + c\dot{h} + kh^3 - Mg = 0$$

Step 3. Define the slopes Δ:
$$\Delta\ddot{h} = \ddot{h} - \bar{\ddot{h}}$$

Step 4. Calculate the partial derivatives of f at the steady state for all the variables:
$$\frac{\partial f}{\partial \ddot{h}} = M\Delta\ddot{h}$$

Step 5. Establish a linear equation using the slopes solving for the highest differential order:
$$\Delta\ddot{h} = -\frac{c}{M}\Delta\dot{h} - \frac{3k(\bar{h})^2}{M}\Delta h$$

ODE diagram



VI. Controllers

Analog controller

- Processes continuous signals using analog components, e.g. resistors, capacitors, and operational amplifiers
- Control signal: continuous voltage or current
- Main feature: fast real-time response with simple control logic

Digital controller

- Uses A/D and D/A converters → discretization and delays
- Slightly reduced dynamic performance due to discretization
- Flexible, precise, scalable, and easy to maintain
- Compact and cost-effective hardware
- Supports multitasking, noise filtering, and advanced algorithms

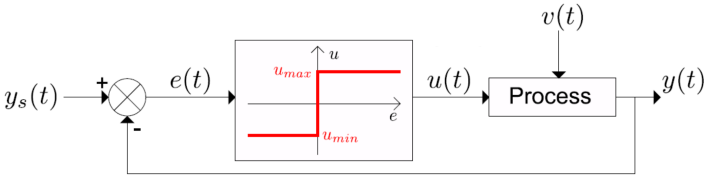
Implementation platforms

- IPCs: complex real-time industrial control
- MCUs: simple low-cost embedded control
- PLCs: robust automation in harsh environments
- DSPs: fast real-time signal calculations
- FPGAs: ultra-fast parallel control
- SBCs: prototyping and low-cost control
- Cloud: remote monitoring, control, and data analysis

Architecture and communication

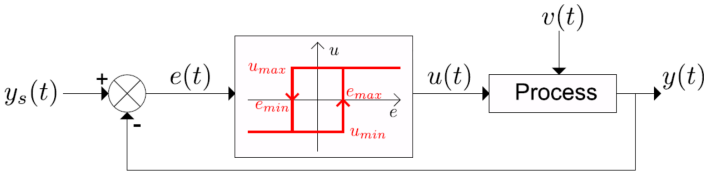
- Centralized control: one unit handles processing, conversion, inputs, and outputs; simple but less scalable
- Fieldbus communication: distributed devices communicate via industrial networks; scalable and modular
- Examples: ProfiNET, CAN/CANopen, EtherCAT

2-point controller



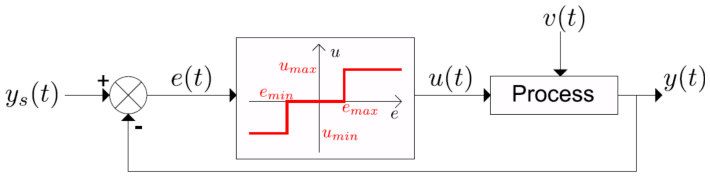
$$\begin{cases} u = u_{max} & \text{if } e > 0 \\ u = u_{min} & \text{if } e < 0 \end{cases}$$

2-point controller with hysteresis



$$\begin{cases} u = u_{max} & \text{if } e > e_{max} \\ u = u_{max} & \text{if } e_{min} < e < e_{max} \text{ and } \dot{e} < 0 \\ u = u_{min} & \text{if } e_{min} < e < e_{max} \text{ and } \dot{e} > 0 \\ u = u_{min} & \text{if } e < e_{min} \end{cases}$$

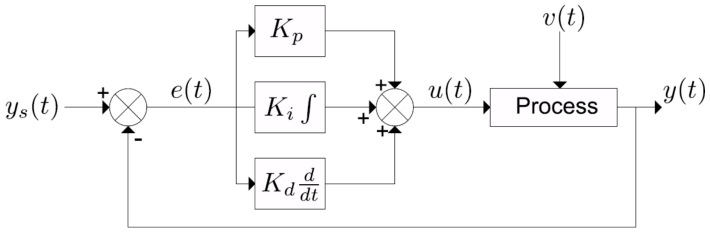
3-point controller



$$\begin{cases} u = u_{max} & \text{if } e > e_{max} \\ u = 0 & \text{if } e_{min} < e < e_{max} \\ u = u_{min} & \text{if } e < e_{min} \end{cases}$$

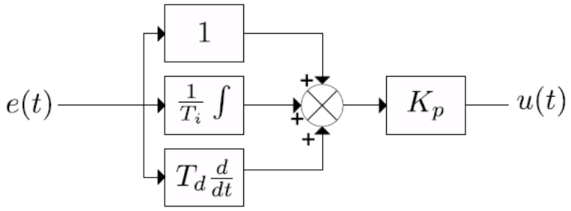
Proportional-Integral-Derivative (PID) controller

Mathematical parametrization



$$u(t) = K_p(e(t)) + K_i \int_0^t e(\tau)\, d\tau + K_d \frac{de(t)}{dt}$$

Technical usual parametrization



$$u(t) = K_p \left(e(t) + \frac{1}{T} \int_0^t e(\tau)\, d\tau + T_d \frac{de(t)}{dt} \right)$$

Controller gain	Integral time	Derivative time
K_p	$T_i = \frac{K_p}{K_i}$	$T_d = \frac{K_d}{K_p}$

PID principle

Proportional term (P)

- Immediate correction proportional to current error
- Reduces rise time
- Cannot remove steady-state error alone
- Too high → oscillation/overshoot

Integral term (I)

- Eliminates steady-state error
- Slow reaction
- Large integral action → overshoot (integral wind-up)

Derivative term (D)

- Predictive action: reacts to error rate of change
- Reduces overshoot and oscillations
- Sensitive to noise

PI controller

- Faster than pure I
- Deletes steady-state error
- Mild overshoot

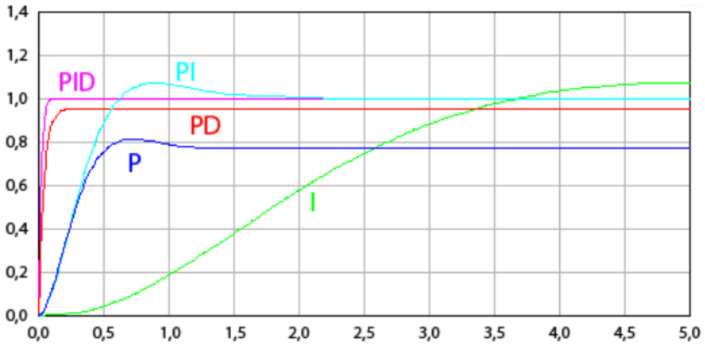
PD controller

- Fast response
- Low overshoot
- Still has steady-state error

PDI controller

- Fast rise time
- No steady-state error
- Limited overshoot
- Best compromise between accuracy + stability

Graphical representation

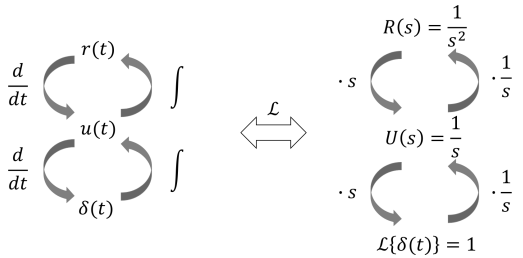


VII. Laplace transform

Laplace transform

The Laplace transform is defined for $t \geq 0$ as

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t) \cdot e^{-st}\, dt$$



Laplace transform of some functions

	$f(t),\ t \geq 0$	$F(s) = \mathcal{L}\{f(t)\}$
Constant	$1 \quad ; \quad a$	$\frac{1}{s} \quad ; \quad \frac{a}{s}$
Step function	$t \quad ; \quad at$	$\frac{1}{s^2} \quad ; \quad \frac{a}{s^2}$
Derivative	$\frac{df(t)}{dt}$	$sF(s) - f(0)$
Integral	$\int_0^t f(\tau)\, d\tau$	$\frac{1}{s}F(s)$
Initial value	$\lim_{t\to 0} f(t)$	$\lim_{s\to\infty} sF(s)$
Steady-state value	$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$
Delayed input	$f(t - t_0)$	$\exp(-s \cdot t_0)F(s)$
Exponential	$\exp(-at)$	$\frac{1}{s+a}$

Benefits of analysis in the s-domain

- Converts differential equations into algebraic equations
- Defines direct input-output relation using transfer functions
- Enables frequency response analysis
- Simplifies stability and system behavior analysis
- Supports structured open-loop and closed-loop analysis
- Makes transfer functions easier to manipulate

VIII. Transfer functions

$$sY(s) + C(s)Y(s) = G(s)u(s) \Longrightarrow Y(s)\,[s + C(s)] = G(s)u(s)$$

$$H_{Y,U}(s) = \frac{Y(s)}{U(s)} = \frac{C(s)}{s + G(s)} = \frac{K_Y}{1 + \tau_Y \cdot s}$$

where K_Y is the gain and τ_Y is the time constant.

Transfer function of the PID controller

ODE of the PID controller:

$$u(t) = K_p \left(e(t) + \frac{1}{T_i} \int_0^t e(\tau)\, d\tau + T_d \dot{e}(t) \right)$$

Transfer function:

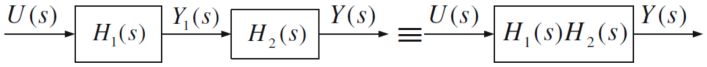
$$U(s) = K_p \left(E(s) + \frac{1}{T_i} \frac{E(s)}{s} + T_d s E(s) \right)$$

$$U(s) = E(s) \left(K_p \frac{T_i T_d s^2 + T_i s + 1}{T_i s} \right)$$

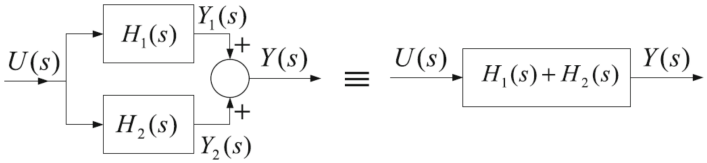
$$\boxed{C(s) = \frac{U(s)}{E(s)} = \left(K_p \frac{T_i T_d s^2 + T_i s + 1}{T_i s} \right)}$$

Block diagram manipulation

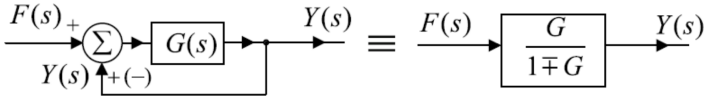
Serial connection



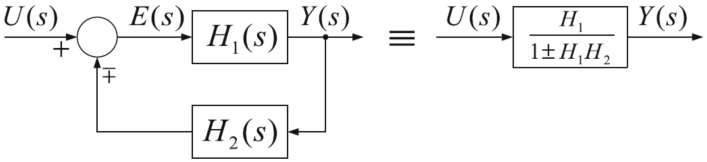
Parallel connection



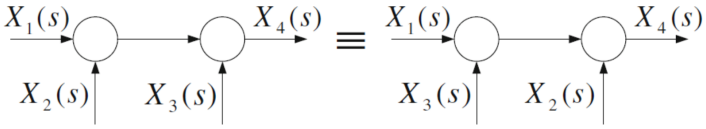
Feedback loop 1



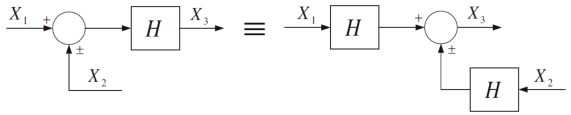
Feedback loop 2



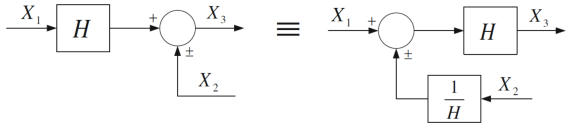
Summation points



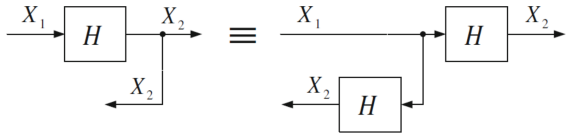
Relocation of a summation 1



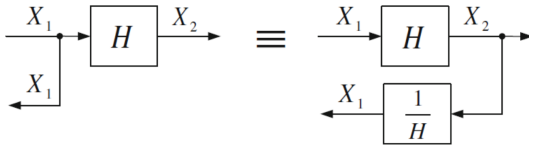
Relocation of a summation 2



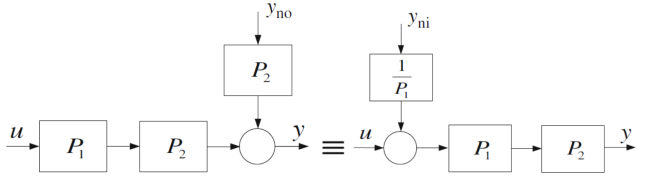
Relocation of a junction 1



Relocation of a junction 2

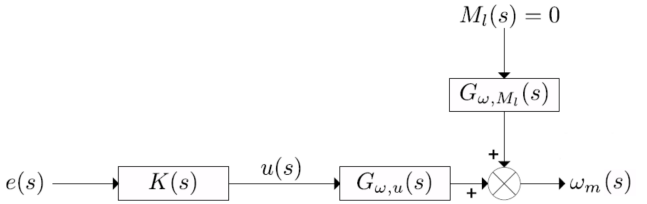


Relocation of a disturbance



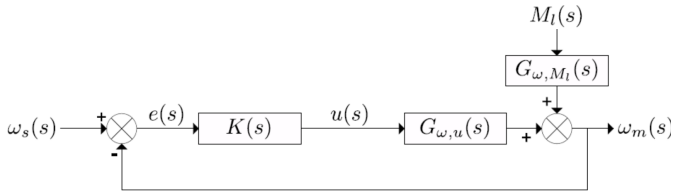
Systems transfer function

Reference open-loop transfer function



$$L(s) = \frac{\omega_m(s)}{e(s)} = K(s)G_{\omega,u}(s)$$

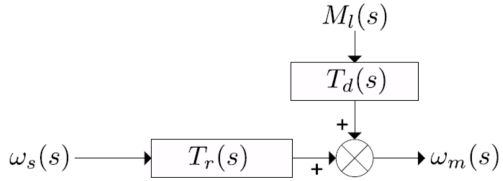
Closed-loop transfer function



$$T_r(s) = \frac{\omega_m(s)}{\omega_s(s)} = \frac{K(s)G_{\omega,u}(s)}{1 + K(s)G_{\omega,u}(s)}$$

$$T_d(s) = \frac{\omega_m(s)}{M_l(s)} = \frac{G_{\omega,M_l}(s)}{1 + K(s)G_{\omega,u}(s)}$$

Controlled system response



$$\omega_m(s) = T_r(s)\omega_s(s) + T_d(s)M_l(s)$$

Zeros and Poles

A system is described by its transfer function:

$$G(s) = \frac{N(s)}{D(s)}$$

- Zeros: roots of $N(s)$
- Poles: roots of $D(s)$

Stability

Bounded input gives bounded output:

$$|u(t)| < \infty \Longrightarrow |y(t)| < \infty$$

Pole conditions:
- $Re < 0$: stable
- Any $Re > 0$: unstable
- Single pole with $Re = 0$: marginally stable
- Repeated poles with $Re = 0$: unstable

Causality

Output depends only on present and past inputs:
- $\deg(N) < \deg(D)$: strictly causal
- $\deg(N) = \deg(D)$: causal
- $\deg(N) > \deg(D)$: non-causal

Oscillations

Oscillations occur with complex poles:
- Real poles only: no oscillation
- $j \pm (Re = 0)$: sustained oscillation
- $j \pm (Re < 0)$: damped oscillation
- $j \pm (Re > 0)$: increasing oscillation

Final value theorem (steady-state)

$$t \to \infty \overset{\mathcal{L}}{\Longleftrightarrow} s \to 0 \quad ; \quad \boxed{\lim_{t\to\infty} y(t) = \lim_{s\to 0} s \cdot Y(s)}$$

Unit step final value

$$Y(s) = G(s)U(S) = G(s) \cdot \frac{1}{s}$$

$$\lim_{s\to 0} s \cdot Y(s) = \lim_{s\to 0} s \cdot \frac{1}{s} \cdot G(s) = \lim_{s\to 0} G(s) = G(s=0)$$

Initial value theorem

$$t \to 0 \overset{\mathcal{L}}{\Longleftrightarrow} s \to \infty \quad ; \quad \boxed{\lim_{t\to 0} y(t) = \lim_{s\to\infty} s \cdot Y(s)}$$

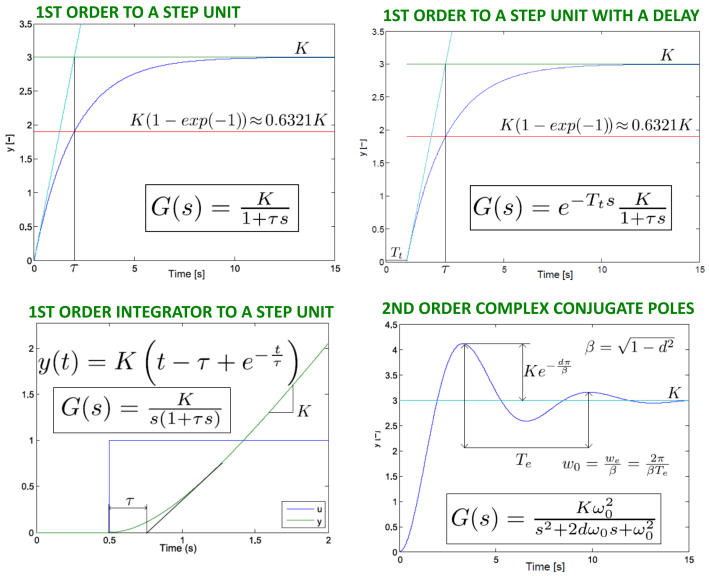
Unit step final value

$$Y(s) = G(s)U(S) = G(s) \cdot \frac{1}{s}$$

$$\lim_{s\to\infty} s \cdot Y(s) = \lim_{s\to\infty} s \cdot \frac{1}{s} \cdot G(s) = \lim_{s\to\infty} G(s)$$

IX. First- and second-order responses

Type of lag responses



X. Controller development

Kuhn method

The process:

$$G(s) = K\frac{(1+\tau_{d,1}s)(1+\tau_{d,2}s)...(1+\tau_{d,m}s)}{(1+\tau_1 s)(1+\tau_2 s)...(1+\tau_n s)}$$

Controller parameters:

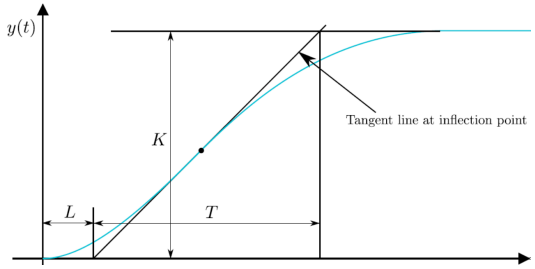
$$T_{\text{sum}} = T_t + \sum_{i=1}^{n}\tau_i - \sum_{j=1}^{m}\tau_{d,j}$$

Controller type	K_p	T_i	T_d
PI	$\frac{1}{K}$	$0.7 \cdot T_{\text{sum}}$	–
PID	$\frac{2}{K}$	$0.8 \cdot T_{\text{sum}}$	$0.194 \cdot T_{\text{sum}}$

Transfer function:

$$C(s) = K_p\left(1 + \frac{1}{T_i s}\right) = K_p\frac{T_i s + 1}{T_i s}$$

Ziegler-Nichols tuning method



First-order lag process with time delay:

$$G(s) = \frac{K}{1+\tau s}e^{-T_t s}$$

Controller parameters:

$$a = \frac{1}{K}\frac{\tau}{T_t} \quad ; \quad L = T_t$$

Controller type	K_p	T_i	T_d
P	a	–	–
PI	$0.9 \cdot a$	$3 \cdot L$	–
PID	$1.2 \cdot a$	$2 \cdot L$	$0.5 \cdot L$

Transfer function:

$$\text{P:} \quad C(s) = K_p = a$$

$$\text{PI:} \quad C(s) = K_p\left(1 + \frac{1}{T_i s}\right) = 0.9 \cdot a\left(1 + \frac{1}{3Ls}\right) = 0.9 \cdot a\frac{3Ls+1}{3Ls}$$

$$\text{PID:} \quad C(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) = 1.2 \cdot a\left(1 + \frac{1}{2Ls} + 0.5Ls\right) = 1$$

True/False

1. Time discretization is called sampling. **T**
2. From a system-modelling point of view, springs and capacitors are analogous as elements in mechanical and electrical systems, respectively. **T**
3. A stable system must exhibit a bounded/limited output for a bounded/limited input. **T**
4. Feedback control systems can feed sensor noise into the system. **T**
5. The integral is represented by s in the s-domain. **F**; integration is represented by $\frac{1}{s}$.
6. A system modelled by $\ddot{y}(t)+\dot{y}(t)=u(t)$ is time-variant. **F**; it is time-invariant.
7. Two first-order systems in series result in a second-order system. **T**
8. High damping coefficient results in oscillatory response of a second-order system. **F**; low damping causes oscillations.
9. Adding a real derivative action (D) to a PI controller strongly decreases oscillations, as well as overshoots. **T**
10. In a PID controller, an increase in the derivative gain affects the steady-state error. **F**; steady-state error is mainly affected by integral action.
11. A process modelled by $3\dot{y}(t)-4y^3(t)=2u(t)$ is linear. **F**
12. A process modelled by $y(t)=t^2x(t)$ is stable. **F**; a bounded input can give an unbounded output.
13. While actuators, sensors and physical processes are in the lowest level of the automation pyramid, the top level represents business management. **T**
14. A parallel combination of two systems represented by $A(s)$ and $B(s)$ results in $H(s)=A(s)\cdot B(s)$. **F**; parallel transfer functions add.
15. The command signal of controller C is equal to $C(e(t))$, with $e(t)$ being the tracking error. **T**
16. First-order systems typically exhibit faster responses than comparable higher-order systems. **T**
17. Feedforward can be used to improve reference tracking and disturbance rejection abilities of a system. **T**
18. Two complex conjugate poles lead to an oscillating behavior of systems. **T**
19. The Laplace transform of the delta function (unit impulse) is $\frac{1}{s}$. **F**; $\mathcal{L}\{\delta(t)\}=1$.
20. The ODE diagram can be drawn for non-linear processes. **T**
21. A process modelled by $\dot{y}(t)=y(t)+u(t)$ is unstable. **T**
22. A process is stable if all poles of its transfer function have negative real parts. **T**
23. The step answer of a first-order lag oscillates. **F**
24. The ODE diagram can be drawn for a non-linear process. **T**
25. Nowadays, most controllers are implemented in a digital way. **T**
26. An instable process can be stabilized with an open-loop controller. **F**
27. A good disturbance rejection behavior means that the tracking error is small for reference signals of interest. **F**; this describes reference tracking.
28. Two complex conjugate poles lead to an oscillating behavior. **T**
29. A process, stable when uncontrolled, remains necessarily stable once closed-loop controlled for any controller. **F**
30. An integrator is needed in a controller to improve the steady-state behavior. **T**
31. Measurement noises are amplified by the derivative component of a controller. **T**
32. An integrator is good to make an unstable process stable. **F**
33. A simple household bread toaster is an example of an open-loop control system. **T**
34. Sensors deliver accurate measurement data of the process output and do not possess any internal noise or inaccuracies. **F**
35. Systems are the physical carriers of information in communication, measurement, and control processes. **F**; signals are the information carriers.
36. The automation pyramid represents the hierarchical structure of industrial automation systems, from field levels at the bottom to enterprise levels at the top. **T**
37. Feedforward can be used to improve reference tracking and disturbance rejection abilities of a system. **T**
38. A process modelled by $\dot{y}(t)-3y^2(t)-2u(t)=0$ is linear. **F**
39. A process modelled by $2\dot{y}(t)-\sqrt{3}y(t)=\ln(2)\cdot u(t)$ is linear. **T**
40. The unit ramp signal is the first derivative of the unit step signal. **F**; the derivative of the ramp is the step.
41. Derivation in the time domain is represented by $\frac{1}{s}$ in the Laplace domain. **F**; differentiation is represented by s.
42. Sampling an analog signal leads to time discretization. **T**
43. Digitalization involves sampling in time and quantization of the amplitude. **T**
44. Integration in the time domain is represented by s in the Laplace domain. **F**; integration is represented by $\frac{1}{s}$.
45. The output of a linear, time-invariant system with impulse response $h(t)$ to an input $x(t)$ is represented in the Laplace domain by $Y(s)=X(s)\cdot H(s)$. **T**
46. A parallel combination of two systems represented by $A(s)$ and $B(s)$ results in $H(s)=A(s)\cdot B(s)$. **F**; parallel transfer functions add.
47. If $H(s)$ were the system transfer function, then $\lim_{s\to 0} H(s)$ would represent the steady-state behavior of the system. **T**
48. Two complex conjugate poles lead to an oscillating behavior of systems. **T**

Components of a medical infusion pump system

Reference signal / setpoint
Set / desired concentration of medication in the patient's bloodstream, set by the nurse. It specifies the desired concentration of medication that the infusion pump system aims to maintain, as prescribed by the healthcare provider.

Controller
Infusion Pump System. It monitors the measured concentration of medication in the patient's bloodstream and adjusts the infusion rate to maintain the set concentration. The controller acts as the controller in the closed-loop system. It compares the measured medication concentration (feedback) with the set concentration (reference signal) and adjusts the infusion rate accordingly to maintain the desired concentration.

Process
Infusion Pump, i.e., the physical system (infusion pump) that delivers the medication into the patient's bloodstream based on the control signal from the infusion pump system. The process represents the physical mechanism (infusion pump) that controls the flow rate of medication into the patient's bloodstream based on the control signal from the infusion pump system.

Process output
Patient's current medication concentration. That is, the actual concentration of medication in the patient's bloodstream, continuously monitored and compared to the set concentration. The actual output of the controlled system is the concentration of medication in the patient's bloodstream. This output is continuously monitored and compared to the set concentration.

Disturbances
Factors such as changes in patient's metabolism, variations in blood flow, or external factors affecting medication absorption. Disturbances are external and internal factors that can affect the medication concentration in the patient's bloodstream, influencing the effectiveness of treatment.

Feedback signal
Measured medication concentration. That is, the actual concentration of medication in the patient's bloodstream, measured by sensors and fed back to the infusion pump system to adjust the infusion rate. The actual concentration of medication in the patient's bloodstream is measured, providing real-time feedback to the infusion pump system to adjust the infusion rate in response to changes.

Laplace transform

Derive $F(s)$ of the following time-domain signal:

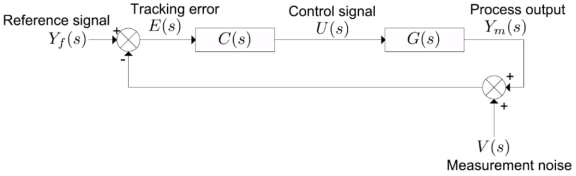
$$f(t)=\frac{1}{4}e^{-5t}\cdot u(t)$$

$$F(s)=\int_0^\infty f(t)e^{-st}\,dt=\int_0^\infty \frac{1}{4}e^{-5t}\cdot u(t)e^{-st}\,dt$$

$$\frac{1}{4}\int_0^\infty e^{-(5+s)t}\,dt=\frac{1}{4}\left[\frac{e^{-(5+s)t}}{-(5+s)}\right]_0^\infty$$

$$\frac{-1}{4(5+s)}\,[0-1]=\frac{1}{4(5+s)},\ \forall Re\{s\}>-5$$

Closed-loop controller



The unique disturbance is the measurement noise $V(s)$.
The process $G(s)$ is modelled by the following Laplace transfer function:

$$G(s)=\frac{K}{(1+T_1s)(1+T_2s)}$$

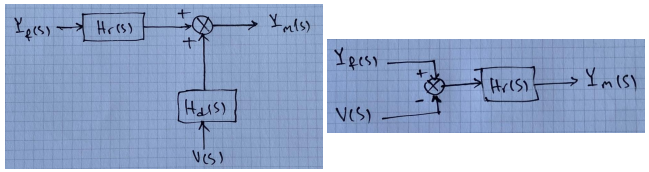
with

$$K=40$$
$$T_1=0.02$$
$$T_2=0.005$$

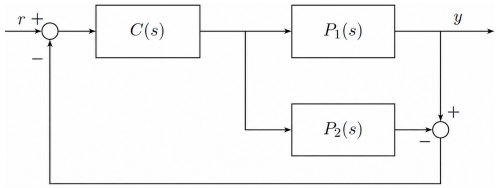
a. [3 points] Calculate the poles and zeros of $G(s)$.
b. [2 points] Is the uncontrolled process stable? Justify your answer.
c. [2 points] Is the uncontrolled process oscillating? Justify your answer.
d. [1 point] A P controller is introduced. What is the generic Laplace transfer function of thi controller (i.e., $C(s)$ =?)?
e. [2 points] Derive the open-loop transfer function $L(s)=C(s)\cdot G(s)$.
f. [3 points] Derive the reference tracking closed-loop transfer function $H_r(s)=\frac{Y_m(s)}{Y_f(s)}$.
g. [3 points] Derive the disturbance rejection closed-loop transfer function $H_d(s)=\frac{Y_m(s)}{V(s)}$.
h. [2 points] Draw the block diagram of the controlled system using $H_r(s)$ and $H_d(s)$.

a. Zeros: $G(s)=0\Rightarrow$ no zeros
Poles: $(1+T_1s)(1+T_2s)=0\Rightarrow s_1=\frac{-1}{T_1}=-50;\ s_2=\frac{-1}{T_2}=-200$
b. Stable, because the real-part of all poles are negative
c. No. There are no complex-conjugate poles
d. $C(s)=K_p$
e. $L(s)=C(s)G(s)=K_p\frac{K}{(1+T_1s)(1+T_2s)}=\frac{40K_p}{10^{-4}s^2+0.025s+1}$
f. $H_r(s)=\frac{L(s)}{1+L(s)}=\frac{\frac{40K_p}{20^{-4}s^2+0.025s+1}}{1+\frac{40K_p}{10^{-4}s^2+0.025s+1}}=\frac{40K_p}{10^{-4}s^2+0.025s+1+40K_p}$
g. $H_d(s)=L(s)\cdot[-v(s)-Y_m(s)]\Rightarrow Y_m(s)\,[1+L(s)]=-v(s)L(s)$
$\Rightarrow\frac{-L(s)}{1+L(s)}=\frac{-40K_p}{10^{-4}s^2+0.025s+1+40K_p}$

h.



Closed-loop block diagram example

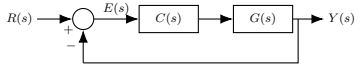


With $P_1(s) = \dfrac{3}{s-1}$, $P_2(s) = \dfrac{1}{s-1}$ and $C(s) = 2$, calculate the transfer function $G(s) = \frac{Y(s)}{R(s)}$

$$u(s) = C(s)e(s), \qquad Y(s) = P_1(s)u(s)$$
$$e(s) = R(s) - (P_1(s)u(s) - P_2(s)u(s)) = R(s) - (P_1(s) - P_2(s))\,C(s)e(s)$$
$$e(s)\,[1 + C(s)\,(P_1(s) - P_2(s))] = R(s)$$
$$G(s) = \frac{Y(s)}{R(s)} = \frac{C(s)P_1(s)}{1 + C(s)\,(P_1(s) - P_2(s))}$$
$$= \frac{2 \cdot \frac{3}{s-1}}{1 + 2\left(\frac{3}{s-1} - \frac{1}{s-1}\right)} = \frac{\frac{6}{s-1}}{\frac{s+3}{s-1}} = \boxed{\frac{6}{s+3}}$$

Steady-state error from block diagram

Question: find the smallest positive k_p such that the unity-feedback system is stable and, for a unit step $r(t)$, $\lim_{t\to\infty} |e(t)| \le 0.1$.



$$C(s) = k_p, \qquad G(s) = \frac{1}{1 + 10s}$$

Unity negative feedback:

$$E(s) = R(s) - Y(s), \qquad Y(s) = C(s)G(s)E(s)$$
$$E(s) = \frac{R(s)}{1 + C(s)G(s)} = \frac{R(s)(1 + 10s)}{1 + 10s + k_p}$$

For a unit step $R(s) = \frac{1}{s}$:

$$e_\infty = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{1}{1 + k_p}$$
$$|e_\infty| \le 0.1 \Rightarrow \frac{1}{1 + k_p} \le 0.1 \Rightarrow k_p \ge 9$$

Closed-loop pole:

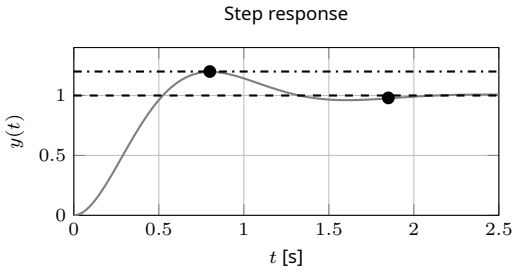
$$1 + 10s + k_p = 0 \Rightarrow s = -\frac{1 + k_p}{10}$$

Stable for $k_p > -1$, therefore the smallest positive gain satisfying both conditions is

$$\boxed{k_p = 9}$$

Order and free integrator from step response

Proportional-control negative feedback, unit step input. The open-loop transfer function $L(s)$ is stated to be 2nd order and without zeros.



a) Question: Use the graph to give two arguments that $L(s)$ is at least 2nd order.

- Overshoot and damped oscillation imply complex conjugate closed-loop poles. A 1st order system cannot oscillate or overshoot without zeros.
- The response starts with zero initial slope, which is typical for a strictly proper system with relative degree ≥ 2.

b) Question: Does the graph justify that $L(s)$ contains a free integrator?

$$T(s) = \frac{Y(s)}{R(s)} = \frac{L(s)}{1 + L(s)}, \qquad y_\infty = T(0)$$

The graph settles at $y_\infty \approx 1$ for a unit step, so the steady-state error is approximately zero:

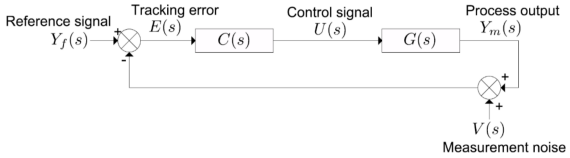
$$e_\infty = 1 - y_\infty \approx 0$$

For unity negative feedback this requires $L(0) \to \infty$, which is obtained when $L(s)$ contains a free integrator:

$$\boxed{L(s) \text{ contains a free integrator}}$$

Closed loop with measurement noise

Question: solve the following closed-loop process with measurement noise $V(s)$, process $G(s)$, and controller $C(s)$.



$$G(s) = \frac{K}{(1 + T_1 s)(1 + T_2 s)}, \qquad K = 30,\ T_1 = 0.01,\ T_2 = 0.001$$

a. **Calculate the poles and zeros of $G(s)$.**
No zeros.
Poles: $1 + T_1 s = 0,\ 1 + T_2 s = 0 \Rightarrow \boxed{s_1 = -100,\ s_2 = -1000}$.

b. **Is this process stable? Justify.**
Stable, because all poles have negative real part.

c. **A P controller is introduced. What is its generic transfer function?**

$$\boxed{C(s) = k_p}$$

d. **Derive the open-loop transfer function $L(s)$.**

$$\boxed{L(s) = C(s)G(s) = \frac{30k_p}{(1 + 0.01s)(1 + 0.001s)} = \frac{30k_p}{10^{-5}s^2 + 0.011s + 1}}$$

e. **Derive the reference tracking closed-loop transfer function.**
With $V = 0$:

$$\boxed{\frac{Y_m(s)}{Y_f(s)} = \frac{L(s)}{1 + L(s)} = \frac{30k_p}{10^{-5}s^2 + 0.011s + 1 + 30k_p}}$$

f. **Derive the disturbance rejection closed-loop transfer function.**
With $Y_f = 0$:

$$E(s) = Y_f(s) - Y_m(s) - V(s), \quad Y_m(s) = L(s)E(s)$$
$$\boxed{\frac{Y_m(s)}{V(s)} = -\frac{L(s)}{1 + L(s)} = -\frac{30k_p}{10^{-5}s^2 + 0.011s + 1 + 30k_p}}$$

g. **Calculate the steady-state error as a function of k_p.**
Unit-step reference and $V = 0$:

$$e_\infty = \lim_{s\to 0} sE(s) = \frac{1}{1 + L(0)} = \boxed{\frac{1}{1 + 30k_p}}$$

h. **Which values of k_p lead to a steady-state error smaller than 5%?**

$$\left|\frac{1}{1 + 30k_p}\right| < 0.05 \quad \text{and stable } (k_p > -1/30) \Rightarrow \boxed{k_p > \frac{19}{30} \approx 0.633}$$

i. **A PID controller is introduced. What is its generic transfer function?**

$$\boxed{C(s) = k_p\left(1 + \frac{1}{T_i s} + T_d s\right)}$$

j. **Choose T_i and T_d so that $L(s)$ simplifies to $\frac{KK_p}{T_i s}$.**
Given $C(s) = K_p(1 + T_d s)\frac{1 + T_i s}{T_i s}$, choose

$$\boxed{T_i = T_1 = 0.01, \qquad T_d = T_2 = 0.001}$$

so that

$$L(s) = C(s)G(s) = K_p(1 + T_2 s)\frac{1 + T_1 s}{T_1 s}\frac{K}{(1 + T_1 s)(1 + T_2 s)} = \boxed{\frac{KK_p}{T_i s}}$$

k. **Derive the reference closed-loop transfer function using these parameters; k_p is free.**

$$\boxed{\frac{Y_m(s)}{Y_f(s)} = \frac{\frac{KK_p}{T_i s}}{1 + \frac{KK_p}{T_i s}} = \frac{KK_p}{T_i s + KK_p} = \frac{30K_p}{0.01s + 30K_p}}$$

l. **Derive the disturbance rejection closed-loop transfer function.**

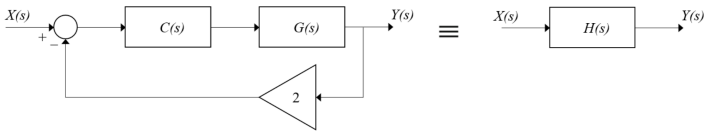
$$\boxed{\frac{Y_m(s)}{V(s)} = -\frac{\frac{KK_p}{T_i s}}{1 + \frac{KK_p}{T_i s}} = -\frac{KK_p}{T_i s + KK_p} = -\frac{30K_p}{0.01s + 30K_p}}$$

m. **Calculate the steady-state error with this new controller.**
With the PID, the free integrator gives

$$e_\infty = \frac{1}{1 + L(0)} = \boxed{0}$$

Equivalent block diagram

Question: the two systems are equivalent. Write $H(s)$ as a function of $C(s)$ and $G(s)$, reporting the algebraic steps.



Let the summing-junction output be $E(s)$. The feedback signal is $2Y(s)$:

$$E(s) = X(s) - 2Y(s)$$
$$Y(s) = C(s)G(s)E(s) = C(s)G(s)\left[X(s) - 2Y(s)\right]$$
$$Y(s) = C(s)G(s)X(s) - 2C(s)G(s)Y(s)$$
$$Y(s)\left[1 + 2C(s)G(s)\right] = C(s)G(s)X(s)$$

Therefore

$$H(s) = \frac{Y(s)}{X(s)} = \boxed{\frac{C(s)G(s)}{1 + 2C(s)G(s)}}$$

First-order process with PI controller

Question: a process is modelled by

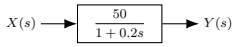
$$G(s) = \frac{Y(s)}{X(s)} = \frac{50}{1 + 0.2s}.$$

a. **Derive the differential equation corresponding to $G(s)$.**

$$\frac{Y(s)}{X(s)} = \frac{50}{1 + 0.2s} \Rightarrow (1 + 0.2s)Y(s) = 50X(s)$$

$$\boxed{y(t) + 0.2\dot{y}(t) = 50x(t)}$$

b. **Draw a block diagram for this process.**



c. **Determine the orders of numerator and denominator of $G(s)$.**

$$\boxed{\deg N(s) = 0, \qquad \deg D(s) = 1}$$

d. **Compute the poles and zeros of this transfer function.**
No zeros. Pole:

$$1 + 0.2s = 0 \Rightarrow \boxed{s = -5}$$

e. **Is this process stable? Explain.**
Yes. The only pole is in the left half-plane: $s = -5 < 0$.

f. **The controller is $C(s) = 0.02\frac{1+0.2s}{s}$. What kind of controller is this?**

$$C(s) = 0.02\frac{1 + 0.2s}{s} = 0.004\frac{1 + 0.2s}{0.2s} = 0.004\left(1 + \frac{1}{0.2s}\right)$$

$$\boxed{\text{PI controller with } k_p = 0.004,\ T_i = 0.2}$$

g. **Derive the open-loop transfer function $L(s)$.**

$$L(s) = C(s)G(s) = 0.02\frac{1 + 0.2s}{s} \cdot \frac{50}{1 + 0.2s} = \boxed{\frac{1}{s}}$$

h. **Derive the reference tracking closed-loop transfer function.**

$$\boxed{\frac{Y(s)}{R(s)} = \frac{L(s)}{1 + L(s)} = \frac{\frac{1}{s}}{1 + \frac{1}{s}} = \frac{1}{s+1}}$$

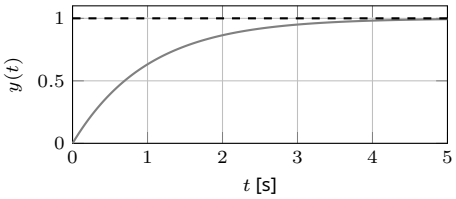
i. **Compute the steady-state error for a unit-step input.**

$$E(s) = \frac{R(s)}{1 + L(s)} = \frac{\frac{1}{s}}{1 + \frac{1}{s}} = \frac{1}{s+1}$$

$$e_\infty = \lim_{s\to 0} sE(s) = \boxed{0}$$

j. **Draw the step answer for a unit-step input.**
Since $\frac{Y(s)}{R(s)} = \frac{1}{s+1}$ and $R(s) = \frac{1}{s}$:

$$Y(s) = \frac{1}{s(s+1)} \Rightarrow \boxed{y(t) = 1 - e^{-t}}$$



First-order lag answer computation

Question: a process can be modelled by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{100}{0.1s + 1}.$$

a. **Derive the differential equation corresponding to $G(s)$.**

$$\frac{Y(s)}{U(s)} = \frac{100}{0.1s + 1} \Rightarrow (0.1s + 1)Y(s) = 100U(s)$$

$$\boxed{0.1\dot{y}(t) + y(t) = 100u(t)}$$

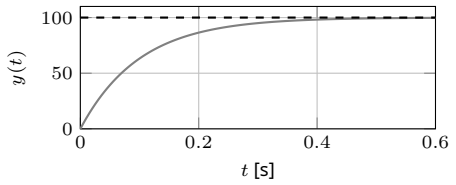
b. **Compute the poles and zeros of this transfer function.**
No zeros. Pole:

$$0.1s + 1 = 0 \Rightarrow \boxed{s = -10}$$

c. **Is this process stable?**
Yes. The only pole is in the left half-plane: $s = -10 < 0$.

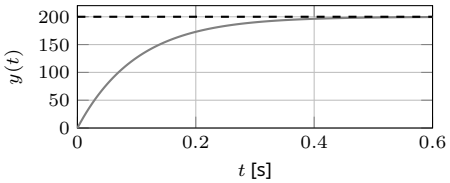
d. **Draw the step answer of this process for a step with amplitude 1.**

$$Y(s) = G(s)\frac{1}{s} = \frac{100}{s(0.1s + 1)} \Rightarrow \boxed{y(t) = 100\left(1 - e^{-10t}\right)}$$



e. **Draw the step answer of this process for a step with amplitude 2.**

$$Y(s) = G(s)\frac{2}{s} = \frac{200}{s(0.1s + 1)} \Rightarrow \boxed{y(t) = 200\left(1 - e^{-10t}\right)}$$



f. **The controller is $C(s) = 0.01\frac{0.1s+1}{s}$. What kind of controller is that?**

$$C(s) = 0.01\frac{0.1s + 1}{s} = 0.001\frac{0.1s + 1}{0.1s} = 0.001\left(1 + \frac{1}{0.1s}\right)$$

$$\boxed{\text{PI controller with } k_p = 0.001,\ T_i = 0.1}$$

g. **Derive the open-loop transfer function $L(s)$.**

$$L(s) = C(s)G(s) = 0.01\frac{0.1s + 1}{s} \cdot \frac{100}{0.1s + 1} = \boxed{\frac{1}{s}}$$

h. **Derive the reference tracking closed-loop transfer function.**

$$\boxed{\frac{Y(s)}{R(s)} = \frac{L(s)}{1 + L(s)} = \frac{\frac{1}{s}}{1 + \frac{1}{s}} = \frac{1}{s+1}}$$

i. **Compute the steady state error.**

$$E(s) = \frac{R(s)}{1 + L(s)} = \frac{\frac{1}{s}}{1 + \frac{1}{s}} = \frac{1}{s+1}$$

$$\boxed{e_\infty = \lim_{s\to 0} sE(s) = 0}$$

j. **Draw the process output if the reference signal is a step with amplitude 1.**

$$Y(s) = \frac{1}{s+1} \cdot \frac{1}{s} = \frac{1}{s(s+1)} \Rightarrow \boxed{y(t) = 1 - e^{-t}}$$

