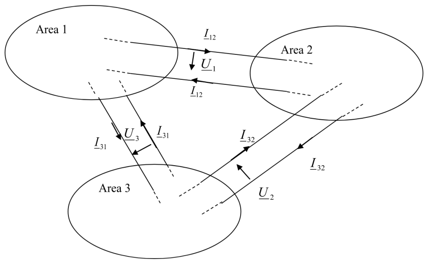
Exercises – Power grid

Exercise 1: Power-flow sign convention

Data:

$$\underline{U}_1 = 76\angle 4^\circ \text{ kV}, \quad \underline{U}_2 = 76\angle 8^\circ \text{ kV}, \quad \underline{U}_3 = 76\angle -11^\circ \text{ kV},$$
$$\underline{I}_{12} = 450\angle -15^\circ \text{ A}, \quad \underline{I}_{31} = 300\angle 18^\circ \text{ A}, \quad \underline{I}_{32} = 500\angle -11^\circ \text{ A}$$



1. Which areas provide active power? How much?
2. Which areas receive reactive power? How much?

$$\boxed{\underline{S} = \underline{U} \cdot \underline{I}^* = \underline{U} + I\angle(-\theta^\circ)}$$

$$\underline{S}_1 = \underline{U}_1 \underline{I}_{31}^* = 76\angle 4^\circ \cdot 450\angle 15^\circ - 76\angle -11^\circ \cdot 300\angle -18^\circ$$
$$= 12.4 + j22.2\,[MW] = 12.4\,[MW] + 22.2\,[MVar]$$
$$\underline{S}_2 = \underline{U}_2 \underline{I}_{32}^* - \underline{U}_1 \underline{I}_{12}^* = 3.59\,[MW] + j1.24\,[MVar]$$
$$\underline{S}_3 = \underline{U}_3 \underline{I}_{31}^* - \underline{U}_2 \underline{I}_{32}^* = -15.99\,[MW] - j23.43\,[MVar]$$

Conventionally, $S, Q > 0$: provider, $S, Q < 0$: receiver

1. $\underline{S}_{1,2}$: provider, 12.4 MW and 3.59 MW
2. $\underline{S}_3$: receiver, 23.43 MVar

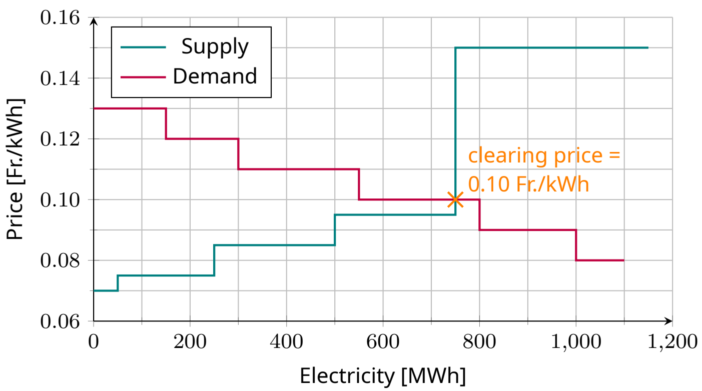
Exercise 2: Merit-order pricing

Sorted list of B1–B6 consumer offers:

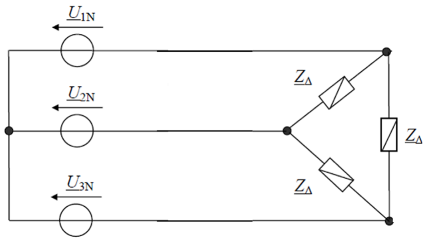
B1	B2	B3	B4	B5	B6
100 MWh 0.08 Fr./kWh	200 MWh 0.09 Fr./kWh	250 MWh 0.10 Fr./kWh	250 MWh 0.11 Fr./kWh	150 MWh 0.12 Fr./kWh	150 MWh 0.13 Fr./kWh

Sorted list of offers of electricity sellers S1–S5:

S1	S2	S3	S4	S5
50 MWh 0.07 Fr./kWh	200 MWh 0.075 Fr./kWh	250 MWh 0.085 Fr./kWh	250 MWh 0.095 Fr./kWh	400 MWh 0.15 Fr./kWh



Exercise 3: Three-phase power



Data:

$$\underline{U}_{1N} = 110\,\text{V}\angle 0^\circ, \quad \underline{U}_{2N} = 230\,\text{V}\angle -120^\circ, \quad \underline{U}_{3N} = 230\,\text{V}\angle 120^\circ,$$
$$\underline{Z}_\Delta = (30 + j25)\,\Omega$$

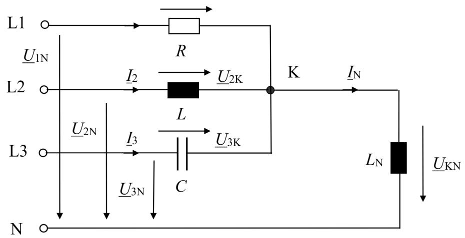
Calculate the complex power $\underline{S}$ and the power factor $\underline{\lambda}$

$$\boxed{\underline{S} = \frac{\sum_i |\underline{U}|_i^2}{\underline{Z}_\Delta^*}}$$

$$\underline{U}_{12} = \underline{U}_{1N} - \underline{U}_{2N} = 110\angle 0^\circ \text{ V} - 230\angle -120^\circ \text{ V} = 300.5\angle 41.52^\circ \text{ V}$$
$$\underline{U}_{23} = \underline{U}_{2N} - \underline{U}_{3N} = 230\angle -120^\circ \text{ V} - 230\angle 120^\circ \text{ V} = 398.5\angle -90^\circ \text{ V}$$
$$\underline{U}_{31} = \underline{U}_{3N} - \underline{U}_{1N} = 230\angle 120^\circ \text{ V} - 110\angle 0^\circ \text{ V} = 300.5\angle 138.48^\circ \text{ V}$$

$$\underline{S} = \frac{|\underline{U}_{12}|^2 + |\underline{U}_{23}|^2 + |\underline{U}_{31}|^2}{\underline{Z}_\Delta^*} = \frac{(300.5\,\text{V})^2 + (300.5\,\text{V})^2 + (398.5\,\text{V})^2}{39.05\angle -39.81^\circ\,\Omega}$$
$$= 6.68 + j5.56\,\text{kVA} = 8.69\angle 39.81^\circ \text{ kW}$$
$$\lambda = \cos(39.81^\circ) = 0.768$$

Exercise 4: 4-wire system



Data:

$$\underline{U}_{1N} = 220\,\text{V}\angle 0^\circ, \quad R = 100\,\Omega,$$
$$\underline{U}_{2N} = 220\,\text{V}\angle -120^\circ, \quad L = 160\,\text{mH},$$
$$\underline{U}_{3N} = 220\,\text{V}\angle 120^\circ, \quad C = 40\,\mu\text{F},$$
$$f = 50\,\text{Hz}, \quad L_N = 500\,\text{mH}.$$

Calculate: $\underline{I}_1$, $\underline{I}_2$, $\underline{I}_3$, $\underline{I}_N$, and $\underline{U}_{1K}$, $\underline{U}_{2K}$, $\underline{U}_{3K}$, $\underline{U}_{KN}$.

$$\boxed{\underline{U}_{KN} = \frac{\underline{Y}_1\underline{U}_{1N} + \underline{Y}_2\underline{U}_{2N} + \underline{Y}_3\underline{U}_{3N}}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3 + \underline{Y}_N} \qquad \underline{U}_{iK} = \underline{U}_{iN} - \underline{U}_{KN} \;;\; \underline{I}_i = \underline{U}_{iK} \cdot \underline{Y}_i}$$

Admittances:

$$\underline{Y}_1 = \underline{Y}_R = \frac{1}{R} = 0.010\angle 0^\circ \text{ S},$$
$$\underline{Y}_2 = \underline{Y}_L = \frac{1}{\omega L}\angle -90^\circ = 0.0199\angle -90^\circ \text{ S},$$
$$\underline{Y}_3 = \underline{Y}_C = \omega C\angle 90^\circ = 0.0126\angle 90^\circ \text{ S},$$
$$\underline{Y}_N = \underline{Y}_L = \frac{1}{\omega L_N}\angle -90^\circ = 0.00637\angle -90^\circ \text{ S}.$$

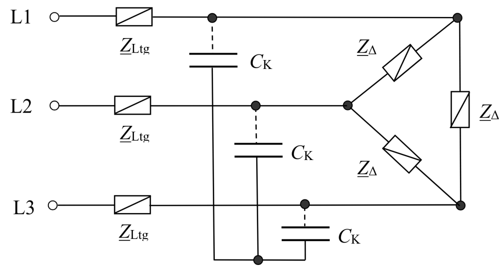
Voltages:

$$\underline{U}_{KN} = \frac{\underline{Y}_1\underline{U}_{1N} + \underline{Y}_2\underline{U}_{2N} + \underline{Y}_3\underline{U}_{3N}}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3 + \underline{Y}_N} = 238.4\angle -137.5^\circ \text{ V},$$
$$\underline{U}_{1K} = \underline{U}_{1N} - \underline{U}_{KN} = 427.2\angle 22^\circ \text{ V},$$
$$\underline{U}_{2K} = \underline{U}_{2N} - \underline{U}_{KN} = 71.9\angle -24^\circ \text{ V},$$
$$\underline{U}_{3K} = \underline{U}_{3N} - \underline{U}_{KN} = 357.8\angle 79.4^\circ \text{ V}.$$

Currents:

$$\underline{I}_1 = \underline{U}_{1K}\underline{Y}_1 = 4.27\angle 22^\circ \text{ A},$$
$$\underline{I}_2 = \underline{U}_{2K}\underline{Y}_2 = 1.43\angle -114^\circ \text{ A},$$
$$\underline{I}_3 = \underline{U}_{3K}\underline{Y}_3 = 4.51\angle 169.4^\circ \text{ A},$$
$$\underline{I}_N = \underline{U}_{KN}\underline{Y}_N = 1.52\angle 132.5^\circ \text{ A}.$$

Exercise 5: Three-phase systems



Data:

$$U_{\text{LL}} = 400V/50Hz \;;\; \underline{Z}_{\text{Ltg}} = (1 + j2)\Omega \;;\; \underline{Z}_\Delta = 30\angle 60^\circ$$

a) Calculate the line currents $\underline{I}$ and the voltages $\underline{U}_\Delta$ across the loads $\underline{Z}_\Delta$ without the capacitor bank connected.
b) Calculate the capacitance C_K of the capacitor bank for reactive current compensation of the loads $\underline{Z}_\Delta$.
c) Determine the line currents $\underline{I}$ in the compensated case.

a) Without compensation:

$$\boxed{\underline{I}_L = \frac{U_{\text{ph}}}{\underline{Z}_{\text{Ltg}} + \underline{Z}_Y} = \frac{U_{\text{LL}}}{\sqrt{3}} \cdot \frac{1}{\underline{Z}_{\text{Ltg}} + \frac{\underline{Z}_\Delta}{3}} \qquad \underline{U}_\Delta = \sqrt{3}\underline{U}_Y = \sqrt{3}\,(\underline{I}_L\underline{Z}_Y) = \sqrt{3}\left(\underline{I}_L \cdot \frac{\underline{Z}_\Delta}{3}\right)}$$

$$\underline{I}_1 = \frac{400V}{\sqrt{3}} \cdot \frac{1}{(1+j2)\Omega + \frac{30\angle 60^\circ}{3}} = 18.88\angle -60.6^\circ A$$

$\underline{I}_2 = \underline{I}_1 - 120^\circ = 18.88\angle 179.4^\circ$
$\underline{I}_3 = \underline{I}_1 + 120^\circ = 18.88\angle 59.4^\circ$

$$\underline{U}_\Delta = \sqrt{3}\left(18.88\angle -60.6^\circ \cdot \frac{30\angle 60^\circ}{3}\right) = 327\angle -0.63^\circ \text{ [V]}$$

$\underline{U}_{12,\Delta} = \underline{U}_\Delta + 30^\circ = 327\angle 29.7^\circ$ [V]
$\underline{U}_{23,\Delta} = \underline{U}_\Delta - 120^\circ = 327\angle -90.63^\circ$ [V]
$\underline{U}_{31,\Delta} = \underline{U}_\Delta + 120^\circ = 327\angle 149.37^\circ$ [V]

$\underline{Z}_\Delta = ¢ = 30\angle 60^\circ \Omega$

b) Compensation capacitance:

$$\underline{Y}_C = j\omega C_k \quad ; \quad \underline{Y}_Y = \frac{1}{\underline{Z}_Y} \quad ; \quad \text{Im}\left(Y_C + Y_Y\right) = 0$$

$\underline{Y}_Y = \frac{3}{\underline{Z}_\Delta} = 0.05 - j0.0866\,\text{S} \to \text{Im}\left(\underline{Y}_Y\right) = -0.0866\,\text{S}$
$\text{Im}(\underline{Y}_C) = -\underline{Y}_Y = 0.0866\,\text{S}$
$0.0866 = \omega C_k \Longrightarrow C_k = \frac{0.0866}{2\pi \cdot 50\,\text{Hz}} = 275.8\,[\mu\text{F}]$

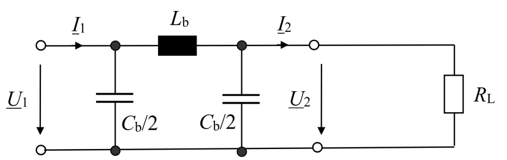
c) With compensation:

$$Y_{\text{load}} = \text{Re}(\underline{Y}_Y) \quad ; \quad Z_{\text{load}} = \frac{1}{Y_{\text{load}}} \quad ; \quad Z_{\text{tot}} = Z_{\text{load}} + \underline{Z}_{\text{Ltg}}$$

$\underline{Z}_{\text{tot}} = 20\Omega + (1+j2)\Omega = 21 + j2\Omega$

$\underline{I}_L = \frac{\underline{U}_{\text{LL}}}{\underline{Z}_{\text{tot}}} \to \underline{I}_1 = \frac{400\,\text{[V]}}{\sqrt{3}} \cdot \frac{1}{21+j2\Omega} = 10.95\angle -5.44^\circ$ [A]
$\underline{I}_2 = \underline{I}_1 - 120^\circ = 10.95\angle -125.44$ [A]
$\underline{I}_3 = \underline{I}_1 + 120^\circ = 10.95\angle 114.56$ [A]

Exercise 6: Surge impedance loading
Equivalent line model of a lossless line with load R_L:

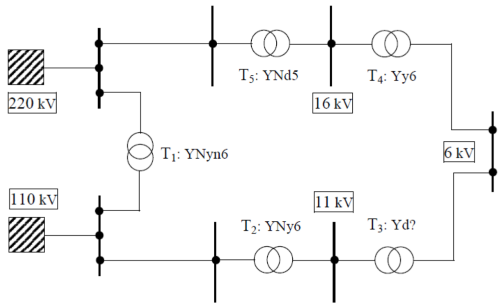


Determine R_L so that the lossless line is optimal operation. Data:
$L'_b = 1.3$ [mH/(km and phase)] ; $C'_b = 9.6$ [nF/(km and phase)]
$l = 250$ [km(length of the line)]

$$Z_C = \sqrt{\frac{L'_b}{C'_b}} = \sqrt{\frac{L_b}{C_b}} \quad ; \quad L_b = L'_b \cdot l \quad ; \quad C_b = C'_b \cdot l$$

$$R_L = Z_C = \sqrt{\frac{1.3\text{E}-3}{9.6\text{E}-9}} = 368\,\Omega$$

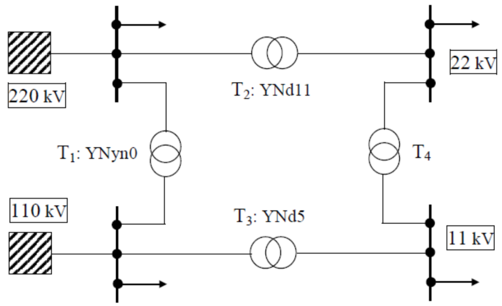
Exercise 7: Transformers 1



Determine the index number of transformer T_3.

$(T5 + T4) \mod 12 = (T_3 + T_2 + T_1) \mod 12$
$11 \mod 12 = (T_3 + 12) \mod 12$
$(11 - \cancel{12}) \mod 12 \equiv T_3 \to -1 \mod 12 \equiv T_3 \to T_3 = 11$

Exercise 8: Transformer 2



Determine a suitable transformer configuration for T_4.

$T_2 + T_4 = T_3 + T_2 \Longrightarrow 11 + T_4 = 5$
$11 + T_4 \equiv 5 \mod 12 \to T_4 = 6$

Suitable transformers: Dd6, Yy6, Dz6.

Exercise 9: Active and reactive power of a synchronous generator
A three-phase synchronous generator (star connected) is built as a non-salient pole machine. It operates at constant voltage and frequency.
Data:
$S_N = 400$ MVA ; $U_N = 20$ kV / 50 Hz ; $X_d = 2$ p.u.
$E = 25$ kV ; $\delta = 30^\circ$

U_N, unless otherwise specified, is line-to-line voltage.

Determine the active and reactive power of the generator. Is the machine capacitive or inductive?

$$P = \frac{U \cdot U_p}{X_d} \sin(\vartheta) \quad ; \quad P_{3p} = 3 \cdot P$$
$$Q = \frac{U \cdot U_p}{X_d} \cos(\vartheta) - \frac{U^2}{X_d} \quad ; \quad Q_{3p} = 3 \cdot Q$$

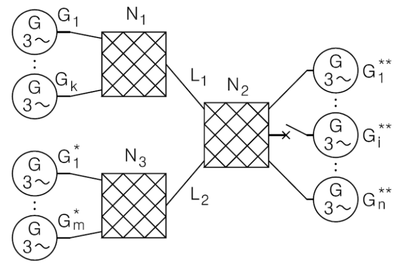
$P = \frac{20\text{E}3\,\text{[V]} \cdot 25\text{E}3\,\text{[V]}}{\sqrt{3} \cdot 2\,\text{[p.u.]}} \sin(30^\circ) = 72.2$ [MW]
$P_{3p} = 3 \cdot 72.2 = 216.6$ [MW]

$Q = \frac{20\text{E}3\,\text{[V]} \cdot 25\text{E}3\,\text{[V]}}{\sqrt{3} \cdot 2\,\text{[p.u.]}} \cos(30^\circ) - \frac{\left(\frac{20\text{E}3\,\text{[V]}}{\sqrt{3}}\right)^2}{2\,\text{p.u.}} = 58.33$ [MVA]
$P_{3p} = 3 \cdot 58.33 = 175$ [MVA]

Exercise 10: Interconnected control areas
Three control areas N_1–N_2–N_3 are connected by L_1 and L_2, initially without power exchange. A 100 MW plant fails in N_2.

$f_N = 50$ Hz ; $K_{N1} = 400$ MW/Hz ; $K_{N2} = K_{N3} = 500$ MW/Hz

a) What frequency will occur in the control areas after the primary controllers have been activated?
b) What power is then exchanged between the areas?
c) What input signals ΔP_R are given to the secondary controllers of the three areas?
d) When will the power exchange on L1 and L2 return to normal?



a) Frequency shift:

$$\Delta P_i = -\sum_i \left(K_{Ni}\right) \Delta f \Longleftrightarrow \Delta f = \frac{\Delta P}{-\left(\sum_i K_{Ni}\right)}$$

$$\Delta f = \frac{100\,\text{[MW]}}{-(400+500+500)\,\text{[MW/Hz]}} = -0.0714\,\text{[Hz]}$$

b) Power exchanged:

$$\Delta P_i = -K_{Ni} \Delta f$$

$\Delta P_1 = -400\,\text{[MW/Hz]} \cdot (-0.0714)\,\text{[Hz]} = 28.56\,\text{[MW]}$
$\Delta P_2 = \Delta P_3 = -500\,\text{[MW/Hz]} \cdot (-0.0714)\,\text{[Hz]} = 35.7\,\text{[MW]}$

c) Secondary controllers:

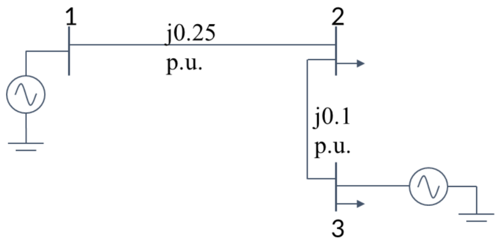
$$\Delta P_{Ri} = \Delta P_i - \Delta P_{Ki}$$

$\Delta P_{R1} = \Delta P_1 + K_{N1}\Delta f = 28.56\,\text{[MW]} - 28.56\,\text{[MW]} = 0\,\text{[MW]}$
$\Delta P_{R2} = \Delta P_3 + K_{N3}\Delta f = 35.7\,\text{[MW]} - 35.7\,\text{[MW]} = 0\,\text{[MW]}$
$\Delta P_{R3} = \Delta P_2 + K_{N2}\Delta f = -\left(\Delta P_1 + \Delta P_3\right) + K_{N2}\Delta f$
$= -(28.56 + 37.5)\,\text{[MW]} + 500 \cdot (-0.0714)\,\text{[MW]} = -100\,\text{[MW]}$

d) Time period: Between 5 to 15 minutes.

Exercise 11: Ybus

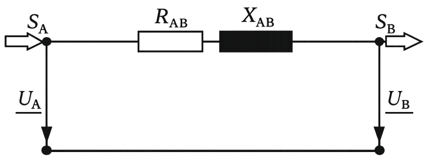
Form the Ybus of the following system:



$$\begin{cases} I_1 = \dfrac{1}{j0.25}(v_1 - v_2) \\ I_2 = \dfrac{1}{j0.1}(v_2 - v_3) + \dfrac{1}{j0.25}(v_2 - v_1) \\ I_3 = \dfrac{1}{j0.1}(v_3 - v_2) \end{cases} \qquad \begin{cases} I_1 = -j4v_1 + j4v_2 \\ I_2 = -j14v_2 + j10v_3 + j4v_1 \\ I_3 = -j10v_3 + j10v_2 \end{cases}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -j4 & j4 & 0 \\ j4 & -j14 & j10 \\ 0 & j10 & -j10 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

Exercise 12a: Two-node system (P_B and Q_B)



Data:

$\underline{U}_A = 400\angle 0^\circ$ [V], $\quad Z_{AB} = 0.1 + j0.1\,[\Omega]$, $\quad \underline{U}_B = 396.3\,e^{j(-0.1774^\circ)}$ [V]

Calculate P_B and Q_B. Is the node inductive or capacitive?

$$Z_{AB} = R_{AB} + jX_{AB} = |Z_{AB}|\,e^{j\theta}$$
$$P_B = \frac{|U_A|\,|U_B|}{|Z_{AB}|}\cos(\theta+\delta) - \frac{|U_B|^2}{|Z_{AB}|}\cos(\theta)$$
$$Q_B = \frac{|U_A|\,|U_B|}{|Z_{AB}|}\sin(\theta+\delta) - \frac{|U_B|^2}{|Z_{AB}|}\sin(\theta)$$

$Z_{AB} = 0.1414e^{j45^\circ}\,[\Omega]$

$$P_B = \frac{400\cdot 396.3}{0.1414}\cos(45^\circ - 0.1774^\circ) - \frac{396.3^2}{0.1414}\cos(45^\circ) = 9815\,[\text{W}]$$

$$Q_B = \frac{400\cdot 396.3}{0.1414}\sin(45^\circ - 0.1774^\circ) - \frac{396.3^2}{0.1414}\sin(45^\circ) = 4908\,[\text{Var}]$$

$Q_B > 0 \rightarrow$ Node B consumes reactive power $\rightarrow$ Inductive system

Exercise 12b: Two-node system (Iterative process for U_B)

Data:

$U_A = 400$ [V] ; $Z_{AB} = 0.1 + j0.1\,[\Omega]$; $P_B = 9815$ [W] ; $Q_B = 4908$ [Var]

Calculate $\underline{U}_B$ (magnitude and phase angle δ).

Step 1) Initialize $i = 0$ and $\underline{U}_B^{(0)} = \underline{U}_A$.

Step 2) Calculate the current: $\underline{I}_B^{(i)} = \left\{\dfrac{P_B + jQ_B}{\underline{U}_B^{(i)}}\right\}^*$.

Step 3) Update the voltage: $\underline{U}_B^{(i+1)} = \underline{U}_A - Z_{AB}\underline{I}_B^{(i)}$.

Step 4) Check convergence: $\left|\left|\underline{U}_B^{(i+1)}\right| - \left|\underline{U}_B^{(i)}\right|\right| < \varepsilon$; $\varepsilon = 1\text{E}-5$.
Otherwise, set $i = i+1$ and repeat from Step 2.

Calculator iteration:

Set Ans $= \underline{U}_A$, then repeatedly enter

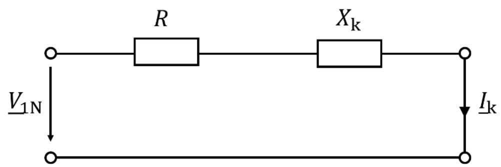
$$\underline{U}_A - (Z_{AB})\,\frac{P_B - jQ_B}{\text{CONJ(Ans)}}$$

until the result no longer changes. The final result is $\underline{U}_B$.

Result:

$\underline{U}_B = 396.28\angle -0.18^\circ$ [V] $= 396.28 - j1.23$ [V]

Exercise 13: Short circuits



Data:

$V_L = 20$ [kV] ; $f = 50$ [Hz] ; $I_k = 300$ [A] ; $R = 12\,[\Omega]$

a) Determine the value of the leakage reactance X_k
b) Calculate the short-circuit current at time $t = 0.03$ s. Assume that the short occurred at $t = 0$ s.

$$i(t) = i_s(t) + i_t(t)$$
$$i_s(t) = \frac{\sqrt{2}V_S}{|Z|}\sin(\omega t + \alpha - \Theta)$$
$$i_t(t) = -i_s(0)\cdot e^{-(R/L)t} = \frac{\sqrt{2}V_S}{|Z|}\sin(\Theta - \alpha)\cdot e^{-(R/L)t}$$
$$|Z| = \sqrt{R^2 + \omega^2 L^2}\angle\left(\Theta = \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

a) Leakage reactance:

$$V_{1N} = \frac{V_L}{\sqrt{3}} \quad ; \quad I_k = \frac{V_{1N}}{|Z|} \Longrightarrow |Z| = \frac{V_L}{\sqrt{3}I_k}$$

$$|Z| = \sqrt{R^2 + X_k^2} \Longrightarrow X_k = \sqrt{|Z|^2 - R^2} = \sqrt{\left(\frac{V_L}{\sqrt{3}I_k}\right)^2 - R^2}$$

$$= \sqrt{\left(\frac{20\text{E}3\,[\text{V}]}{\sqrt{3}\cdot 300\,[\text{A}]}\right)^2 - (12\,[\Omega])^2} = 36.57\,[\Omega]$$

b) Short circuit:

$$X_k = 2\pi f K \Longrightarrow K = \frac{X_k}{2\pi f} = \frac{36.57\,[\Omega]}{2\pi\cdot 50\,[\text{Hz}]} = 0.1164\,[\text{H}]$$

$$\begin{aligned} i_t(0) &= -i_s(0)e^{-(R/K)t} \\ i_t(0.03) &= \sqrt{2}I_k\sin(\Theta)e^{-(12/0.1164)\cdot 0.03} \\ &= \sqrt{2}I_k\sin\left(\arctan\left(\frac{X_k}{R}\right)\right)e^{-(12/0.1164)\cdot 0.03} \\ &= \sqrt{2}\cdot 300\,[\text{A}]\sin\left(\arctan\left(\frac{36.57\,[\Omega]}{12\,[\Omega]}\right)\right)e^{-(12/0.1164)\cdot 0.03} \\ &= 18.29\,[\text{A}] \end{aligned}$$

$$\begin{aligned} i_s(0.03) &= \sqrt{2}I_k\sin\left(2\pi f\cdot 0.03\,[\text{s}] - \arctan\left(\frac{X_k}{R}\right)\right) \\ &= \sqrt{2}\cdot 300\,[\text{A}]\sin\left(2\pi\cdot 50\,[\text{Hz}]\cdot 0.03\,[\text{s}] - \arctan\left(\frac{0.1164\,[\text{H}]}{12\,[\Omega]}\right)\right) \\ &= 403.12\,[\text{A}] \end{aligned}$$

$$i(t) = i_t(t) + i_s(t) = 403.12\,[\text{A}] + 18.29\,[\text{A}] = 421.41\,[\text{A}]$$

Exercise 14: Gas and steam combined cycle power plant (CCGT)

Data:

$P_{\text{th,fuel}} = 600$ [MW] ; $P_{\text{el,GT}} = 240$ [MW] ; $\dot{m}_{\text{exh}} = 500$ [kg/s]

$T_{\text{in}} = 600^\circ$C ; $T_{\text{out}} = 100^\circ$C ; $c_{p,\text{exh}} = 1.1$ [kJ/(kgK)] ; $\eta_{\text{DT}} = 40\%$

Calculate η_{GT}, transferred heat $\dot{Q}_{\text{trans}}$, steam-turbine power $P_{\text{el,DT}}$, total power and total efficiency.

$$\eta_{\text{GT}} = \frac{P_{\text{el,GT}}}{P_{\text{th,fuel}}}$$
$$\dot{Q}_{\text{trans}} = \dot{m}_{\text{exh}}c_{p,\text{exh}}\Delta T$$
$$P_{\text{el,DT}} = \eta_{\text{DT}}\dot{Q}_{\text{trans}}$$
$$P_{\text{el,tot}} = P_{\text{el,GT}} + P_{\text{el,DT}} \quad ; \quad \eta_{\text{tot}} = \frac{P_{\text{el,tot}}}{P_{\text{th,fuel}}}$$

Solution:

$$\eta_{\text{GT}} = \frac{240}{600} = 0.40 = 40\%$$

$$\Delta T = 600°\text{C} - 100°\text{C} = 500\,[\text{K}]$$
$$\dot{Q}_{\text{trans}} = 500 \cdot 1.1 \cdot 500\,[\text{kJ/s}] = 275000\,[\text{kW}] = 275\,[\text{MW}]$$
$$P_{\text{el,DT}} = 0.40 \cdot 275\,[\text{MW}] = 110\,[\text{MW}]$$
$$P_{\text{el,tot}} = 240 + 110 = 350\,[\text{MW}] \quad ; \quad \eta_{\text{tot}} = \frac{350}{600} = 0.583 = 58.3\%$$

Discussion: the exhaust heat adds 110 MW without additional fuel. Lower chimney temperature would increase efficiency, but condensation of acidic components can cause corrosion.

Exercise 15: Fuel consumption of a nuclear power plant

Data:

$$P_{\text{el,net}} = 1010\,[\text{MW}]; \quad \eta_{\text{el}} = 34\%; \quad E_{\text{fission}} \approx 200\,[\text{MeV}]$$
$$M_{\text{U235}} = 235.04\,[\text{g/mol}]; \quad 1\,[\text{MeV}] = 1.602 \cdot 10^{-13}\,[\text{J}]$$

Calculate the required reactor thermal power, fission rate and daily consumed mass of split U-235.

$$P_{\text{th}} = \frac{P_{\text{el,net}}}{\eta_{\text{el}}}$$
$$\dot{N}_{\text{fission}} = \frac{P_{\text{th}}}{E_{\text{fission}}}$$
$$N_{\text{day}} = \dot{N}_{\text{fission}} \cdot 86400 \quad ; \quad n = \frac{N_{\text{day}}}{N_A} \quad ; \quad m = nM_{\text{U235}}$$

Solution:

$$P_{\text{th}} = \frac{1010\,[\text{MW}]}{0.34} = 2970.6\,[\text{MW}]$$
$$E_{\text{fission}} = 200 \cdot 1.602 \cdot 10^{-13}\,[\text{J}] = 3.204 \cdot 10^{-11}\,[\text{J}]$$
$$\dot{N}_{\text{fission}} = \frac{2.9706 \cdot 10^{9}\,[\text{J/s}]}{3.204 \cdot 10^{-11}\,[\text{J}]} = 9.27 \cdot 10^{19}\,[\text{s}^{-1}]$$
$$N_{\text{day}} = 9.27 \cdot 10^{19} \cdot 86400 = 8.01 \cdot 10^{24} \quad ; \quad n = \frac{8.01 \cdot 10^{24}}{6.022 \cdot 10^{23}} = 13.3\,[\text{mol}]$$
$$m = 13.3 \cdot 235.04\,[\text{g}] = 3126\,[\text{g}] \approx 3.13\,[\text{kg/day}]$$

Discussion: only a few kg of actually split U-235 provide the daily energy. Real fuel assemblies contain mostly U-238 and only a few percent fissile U-235, and remain in the reactor for years.

Exercise 16: Hydrothermal geothermal power plant

Data:

$$T_{\text{prod}} = 130°\text{C} \quad ; \quad T_{\text{rein}} = 70°\text{C} \quad ; \quad \dot{m}_{\text{th}} = 80\,[\text{kg/s}]$$
$$c_{p,\text{water}} = 4.2\,[\text{kJ/(kgK)}] \quad ; \quad \eta_{\text{ORC}} = 12\% \quad ; \quad E_{\text{house}} = 4000\,[\text{kWh/a}]$$

Assume 8000 operating hours per year. Calculate extracted thermal power, net electrical power and number of supplied households.

$$P_{\text{th}} = \dot{m}_{\text{th}} c_{p,\text{water}} \Delta T$$
$$P_{\text{el,net}} = \eta_{\text{ORC}} P_{\text{th}}$$
$$E_{\text{plant}} = P_{\text{el,net}} t_{\text{op}} \quad ; \quad N_{\text{house}} = \frac{E_{\text{plant}}}{E_{\text{house}}}$$

Solution:

$$\Delta T = 130°\text{C} - 70°\text{C} = 60\,[\text{K}]$$
$$P_{\text{th}} = 80 \cdot 4.2 \cdot 60\,[\text{kJ/s}] = 20160\,[\text{kW}] = 20.16\,[\text{MW}]$$
$$P_{\text{el,net}} = 0.12 \cdot 20.16\,[\text{MW}] = 2.42\,[\text{MW}]$$
$$E_{\text{plant}} = 2.42\,[\text{MW}] \cdot 8000\,[\text{h/a}] = 19360\,[\text{MWh/a}] = 19360000\,[\text{kWh/a}]$$
$$N_{\text{house}} = \frac{19360000}{4000} = 4840$$

Discussion: the electrical efficiency is low because the temperature level is low, but geothermal plants can provide base-load energy. The re-injected 70°C water is still valuable for district heating or CHP; drilling success and mass flow are the main economic risks.

Exercise 17: Hydropower turbine power

Data:

$$\rho = 1000\,[\text{kg/m}^3] \quad ; \quad g = 9.81\,[\text{m/s}^2] \quad ; \quad \eta_T = 85\% \quad ; \quad \eta_G = 95\%$$

Site	Head h	Flow Q
A: high Alps	800 m	15 m^3/s
B: upland river	60 m	120 m^3/s
C: lowland weir	12 m	400 m^3/s

Select turbine type and estimate power. For Site A, check the calculation.

$$P_{\text{hydr}} = \rho g Q h$$
$$P_{\text{el}} = P_{\text{hydr}} \eta_T \eta_G$$

Turbine selection:

Site	Turbine	Diagram estimate
A	Pelton	approx. 100–110 MW
B	Francis	approx. 60–65 MW
C	Kaplan	approx. 40 MW

Site A check:

$$P_{\text{hydr}} = 1000 \cdot 9.81 \cdot 15 \cdot 800 = 117720000\,[\text{W}] = 117.72\,[\text{MW}]$$
$$P_{\text{el}} = 117.72 \cdot 0.85 \cdot 0.95 = 95.06\,[\text{MW}]$$

Losses: hydraulic friction/turbulence, mechanical bearing losses, generator copper/iron losses, transformer and line losses.

Exercise 18: Raising the dams at Lake Grimsel

Data:

$$h_{\text{old}} = 100\,[\text{m}] \quad ; \quad b_{\text{old}} = 800\,[\text{m}] \quad ; \quad L = 4000\,[\text{m}] \quad ; \quad \Delta h = 23\,[\text{m}]$$
$$h_{\text{new}} = 123\,[\text{m}] \quad ; \quad \rho = 1000\,[\text{kg/m}^3] \quad ; \quad h_{\text{eff}} \approx 110\,[\text{m}]$$

Model the valley as an upside-down triangular prism.

$$b(h) = \frac{b_{\text{old}}}{h_{\text{old}}} h$$
$$A(h) = b(h) L$$
$$V = \int_0^h A(h)\,\mathrm{d}h$$
$$\Delta E_{\text{pot}} = \rho \Delta V g h_{\text{eff}}$$

Geometry:

$$b(h) = \frac{800}{100} h = 8h \quad ; \quad A(h) = 8h \cdot 4000 = 32000h$$
$$V_{\text{old}} = \int_0^{100} 32000h\,\mathrm{d}h = 16000h^2\Big|_0^{100} = 160 \cdot 10^6\,[\text{m}^3]$$
$$V_{\text{new}} = 16000 \cdot 123^2 = 242.064 \cdot 10^6\,[\text{m}^3] \approx 242\,[\text{Mio. m}^3]$$
$$\Delta V = V_{\text{new}} - V_{\text{old}} \approx 82\,[\text{Mio. m}^3]$$
$$\frac{\Delta h}{h_{\text{old}}} = 23\% \quad ; \quad \frac{\Delta V}{V_{\text{old}}} = \frac{82}{160} = 51.25\%$$
$$\Delta E_{\text{pot}} = 1000 \cdot 82 \cdot 10^6 \cdot 9.81 \cdot 110 = 8.84 \cdot 10^{13}\,[\text{J}] = 24.55\,[\text{GWh}]$$

Discussion: technically attractive because storage volume grows disproportionately with height. Environmental argument: flooding upper valley areas can destroy rare moor/glacial habitats irreversibly.

Exercise 19: Pumped-storage cycle

Data:

$$D = 600\,[\text{m}] \quad ; \quad h_{\text{stat,min}} = 450\,[\text{m}] \quad ; \quad h_{\text{loss}} = 10\,[\text{m}]$$
$$Q_{\text{pump}} = 50\,[\text{m}^3/\text{s}] \quad ; \quad t_{\text{pump}} = 6\,[\text{h}] \quad ; \quad \eta_{\text{pump}} = 90\%$$
$$Q_{\text{turb}} = 65\,[\text{m}^3/\text{s}] \quad ; \quad \eta_{\text{turb}} = 92\%$$
$$p_{\text{buy}} = 40\,[\text{CHF/MWh}] \quad ; \quad p_{\text{sell}} = 180\,[\text{CHF/MWh}]$$

Calculate pumped water volume, reservoir level rise, mean pumping head, electrical input energy, mean turbine head, generated output energy, turbine operating time, round-trip efficiency and economic profit/loss. Discuss whether the cycle is worthwhile and why pumped storage is system-relevant despite losses.

$$V = Qt \quad ; \quad A = \pi (D/2)^2 \quad ; \quad \Delta h = V/A$$
$$W_{\text{in}} = \frac{\rho g V h_{\text{pump,mean}}}{\eta_{\text{pump}}}$$
$$W_{\text{out}} = \rho g V h_{\text{turb,mean}} \eta_{\text{turb}}$$
$$\eta_{\text{cycle}} = \frac{W_{\text{out}}}{W_{\text{in}}}$$

Pumping:

$$V = 50 \cdot 6 \cdot 3600 = 1080000\,[\text{m}^3]$$
$$A = \pi \cdot 300^2 = 282743\,[\text{m}^2] \quad ; \quad \Delta h = \frac{1080000}{282743} = 3.82\,[\text{m}]$$
$$h_{\text{pump,mean}} = 450 + \frac{3.82}{2} + 10 = 461.91\,[\text{m}]$$
$$W_{\text{in}} = \frac{1000 \cdot 9.81 \cdot 1080000 \cdot 461.91}{0.90} = 5.435 \cdot 10^{12}\,[\text{J}] = 1509.7\,[\text{MWh}]$$

Turbining:

$$h_{\text{turb,mean}} = 450 + \frac{3.82}{2} - 10 = 441.91\,[\text{m}]$$
$$W_{\text{out}} = 1000 \cdot 9.81 \cdot 1080000 \cdot 441.91 \cdot 0.92 = 4.312 \cdot 10^{12}\,[\text{J}] = 1197.8\,[\text{MWh}]$$
$$t_{\text{turb}} = \frac{1080000}{65} = 16615\,[\text{s}] = 4.62\,[\text{h}]$$
$$\eta_{\text{cycle}} = \frac{1197.8}{1509.7} = 79.3\%$$

$$C = 1509.7 \cdot 40 = 60388\,[\text{CHF}] \quad ; \quad R = 1197.8 \cdot 180 = 215604\,[\text{CHF}]$$
$$\text{Profit} = R - C = 155216\,[\text{CHF}]$$

Decision: economically worthwhile for this price spread. System value: fast balancing power, absorption of renewable surplus and firm peak capacity.

Exercise 20: Dynamic grid tariffs

Task: Compare CKW standard tariffs with dynamic grid tariffs and identify which bill components fluctuate.

Facts:

- Standard tariff: fixed or time-of-use price blocks known in advance.
- Dynamic grid tariff: 15-min grid-use price changes according to grid congestion; published day-ahead.
- In basic supply, the energy component is not freely dynamic; dynamic restructuring mainly applies to grid usage.

Component split:

$$p_{\text{tot}} = (p_{\text{energy}} + p_{\text{grid,dyn}} + p_{\text{levies}} + p_{\text{winter}})\,(1 + \text{VAT})$$

Only $p_{\text{grid,dyn}}$ fluctuates in the dynamic grid tariff. Energy, levies, winter reserve and VAT remain static for the tariff calculation.

Dynamic-grid extremes from slides:

$$p_{\text{grid,dyn,min}} = 0.1\,[\text{Rp/kWh}] \quad ; \quad p_{\text{grid,dyn,max}} = 32.0\,[\text{Rp/kWh}]$$
$$\Delta p_{\text{grid}} = 31.9\,[\text{Rp/kWh}]$$

Interpretation: dynamic grid tariffs do not make electricity production cheaper; they create a financial signal to shift flexible loads away from local grid peaks.

Exercise 21: Dynamic sharing coefficient

Data:
$$P_{\text{PV}} = 12\,[\text{kW}] \quad ; \quad L_A = 14\,[\text{kW}] \quad ; \quad L_B = 4\,[\text{kW}]$$

Allocate PV to peers A and B to minimize the highest remaining grid import.

$$\begin{array}{l} \min P_{\max} \\ a_A + a_B = P_{\text{PV}} \quad ; \quad 0 \le a_A \le L_A \quad ; \quad 0 \le a_B \le L_B \\ P_{\max} \ge L_A - a_A \quad ; \quad P_{\max} \ge L_B - a_B \end{array}$$

Solution: optimum equalizes the two remaining grid imports:

$$L_A - a_A = L_B - a_B \quad ; \quad a_A + a_B = 12$$
$$14 - a_A = 4 - a_B \Rightarrow a_A - a_B = 10 \quad ; \quad a_A = 11\,[\text{kW}] \quad ; \quad a_B = 1\,[\text{kW}]$$
$$P_{\text{grid},A} = 14 - 11 = 3\,[\text{kW}] \quad ; \quad P_{\text{grid},B} = 4 - 1 = 3\,[\text{kW}]$$
$$c_A = \frac{11}{12} = 91.7\% \quad ; \quad c_B = \frac{1}{12} = 8.3\%$$

Result: the dynamic allocation shaves the maximum individual grid peak from 14 kW to 3 kW.

Exercise 22: Choose ZEV, vZEV or LEG

Task: For each scenario choose the regulatory framework and allocation key.

Scenario	Type	Allocation key
MFH + supermarket, one property	ZEV	hierarchical or fixed investor-first
5 SFH on same street / LV box	vZEV	hierarchical, then proportional
village: school, bakery, households	LEG	hierarchical political cascade

Reasoning: ZEV is best behind one meter/property and saves all grid fees internally. vZEV avoids digging private cables while allowing nearby low-voltage neighbours to share energy digitally. LEG is needed for municipal scale across several transformer zones and requires a legal entity.

Allocation logic: hierarchical sharing is used when someone has a priority claim, e.g. investor ROI, House A's EV or the mayor's bakery-first policy. Proportional sharing is fairer for remaining energy after priority loads are served.

Exercise 23: Sonnendorf smart-grid memo

Task: Write a short technical memo using at least 8 abbreviations: ZEV, vZEV, LEG, DSO, EMS, AMI, BESS, ToU, cVPP, DR.

Model answer: Sonnendorf should create a **LEG** because the school, bakery and SFHs are spread across the municipality and need to use the public **DSO** grid. The **AMI** smart meters provide 15-min data for allocation and billing. An **EMS** coordinates the school PV, household EV charging and bakery loads. A community **BESS** stores midday surplus and supplies evening loads. **ToU**/dynamic grid tariffs should trigger **DR**: EVs and heat pumps shift away from expensive grid periods. The assets can be aggregated as a **cVPP** to reduce peaks and possibly offer flexibility services. Individual apartment blocks could still use **ZEV**; neighbouring SFHs could use **vZEV**, but the village-wide solution is the LEG.

SW 1: Introduction

Swiss grid levels

Electric power grids are split into voltage levels because the optimal voltage depends on distance and power. High voltage reduces current and losses during transport; lower voltage is safer, cheaper to insulate and easier to switch close to consumers.

Voltage	Purpose
380/220 kV	long-distance transport, import/export, large power plants
~ 110 kV	regional transport, large industry, connection to HV grid
10–40 kV	medium-voltage distribution in cities/regions
0.4 kV	final low-voltage supply for households and small consumers

Choice of grid voltage

Transmission losses

For a given transmitted active power, increasing voltage lowers the current. Since ohmic losses grow with I^2, high voltage is the main technical reason for economical long-distance transmission.

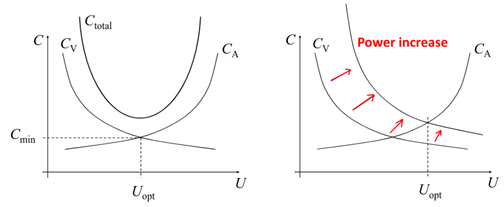
$$P_{\text{trans}} = U_m I \quad ; \quad P_\Omega = RI^2 = R\left(\frac{P_{\text{trans}}}{U_m}\right)^2$$

$$P_{\text{trans}} = \sqrt{3}\,U_L I_L \cos\varphi \quad ; \quad P_\Omega = \frac{P_{\text{trans}}^2 R}{U_L^2 \cos^2\varphi}$$

where: P_{trans} = transmitted active power; P_Ω = resistive loss; U_L = line-to-line RMS voltage.

Economic optimum

Voltage cannot be increased indefinitely: insulation, towers, clearances, switchgear and protection become more expensive. The economical voltage is the minimum of loss cost and investment cost.



$$C_{\text{total}} = C_V + C_A$$

where: C_V = transmission-loss cost; C_A = insulation/infrastructure cost. **Rule of thumb**: economical transmission requires ~ 1 kV/km.

Reliability

$N-1$ criterion

A secure grid is planned with redundancy. After the loss of any single relevant component (line, transformer, generator, busbar section), the remaining system must stay within current, voltage and stability limits.

Consequence: assets are not operated permanently at their thermal limit, because spare capacity is needed after a contingency. $N-1$ improves security but increases investment and operating cost.

SAIDI

SAIDI: average duration of long interruptions per customer and year:

$$\text{SAIDI} = \frac{\sum \text{customer interruption durations}}{\text{number of customers}}$$

Low SAIDI means high supply reliability. It measures interruption duration, not the size of the interrupted power.

Electricity markets

Responsibilities

- **TSO**: operates the transmission grid, keeps frequency stable, procures balancing energy and manages congestion; CH: Swissgrid
- **Balance group**: financially responsible for deviations between scheduled and actual feed-in/withdrawal
- **Schedule**: planned energy exchange per time interval; deviations are settled via balancing mechanisms

Trading horizons

Market	Role
Futures/OTC	long-term price hedging and bilateral contracts
Day-ahead	auction for the next delivery day; main spot reference price
Intraday	correction of forecast errors close to delivery
Balancing	activated by TSO to keep generation and load equal in real time

Merit order

Offers are sorted by increasing marginal cost. Low-cost plants are dispatched first; the last accepted offer sets the market clearing price for all accepted energy in the auction.

$$p_{\text{market}} = \text{price of the last accepted offer}$$

High demand or low renewable feed-in shifts the clearing point to more expensive plants. This is why gas or peak units can set the price even if many low-cost units are producing.

SW 2: Three-Phase Systems

AC elements and phasors

Sinusoidal quantities

RMS values are used because they produce the same heating effect as DC. Phasors replace sinusoidal time functions by complex numbers; this turns differential equations in steady-state AC circuits into algebraic equations.

$$U = \frac{\hat{U}}{\sqrt{2}} \quad ; \quad \omega = 2\pi f \quad ; \quad \underline{U} = U e^{j\varphi_u} = U\angle\varphi_u$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \longleftrightarrow j\omega \quad ; \quad \int \mathrm{d}t \longleftrightarrow \frac{1}{j\omega} \quad ; \quad \underline{U} = \underline{Z}\underline{I} \quad ; \quad \underline{Y} = \frac{1}{\underline{Z}}$$

R, L, C behaviour

Resistance dissipates active power. Inductance stores energy in a magnetic field and makes current lag. Capacitance stores energy in an electric field and makes current lead. Impedance describes opposition to current; admittance is useful when parallel branches or neutral displacement are calculated.

	Impedance	Phase
R	$\underline{Z}_R = R$	u, i in phase
C	$\underline{Z}_C = 1/(j\omega C)$	i leads u by 90°
L	$\underline{Z}_L = j\omega L$	i lags u by 90°

Complex power

Power components

Complex power separates energy conversion from energy oscillation. Active power P is converted into mechanical work, heat or light. Reactive power Q oscillates between source and fields; it increases currents and voltage drops although no net energy is consumed over a period.

$$\underline{S} = \underline{U}\underline{I}^* = P + jQ \quad ; \quad S = UI = \sqrt{P^2 + Q^2}$$

$$P = UI\cos\varphi \quad ; \quad Q = UI\sin\varphi \quad ; \quad \lambda = \cos\varphi = \frac{P}{S}$$

where: $\varphi = \varphi_u - \varphi_i$; $Q > 0$ inductive/lagging; $Q < 0$ capacitive/leading.

Load and generator signs

With the consumer reference direction, positive P and Q mean absorption. With generator convention, positive active power means power delivered to the grid. Always check the current arrow before interpreting the sign.

Case	Typical meaning
lagging current	inductive load, consumes Q
leading current	capacitive behaviour, supplies Q in load convention
over-excited generator	supplies reactive power to the grid
under-excited generator	absorbs reactive power from the grid

Balanced three-phase systems

Phase system

A balanced three-phase system has three equal sinusoidal phase voltages shifted by 120°. The sum of the three phase voltages is zero, so no neutral current flows for a balanced load. The total instantaneous power is constant, which gives smooth torque in machines.

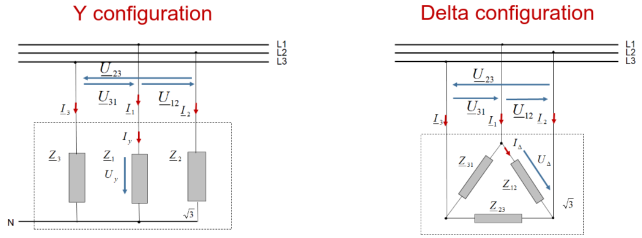
$$\underline{U}_{1N} = U_{\text{ph}}\angle 0^\circ \quad ; \quad \underline{U}_{2N} = U_{\text{ph}}\angle -120^\circ \quad ; \quad \underline{U}_{3N} = U_{\text{ph}}\angle 120^\circ$$

$$\underline{U}_{12} = \underline{U}_{1N} - \underline{U}_{2N} = \sqrt{3}U_{\text{ph}}\angle 30^\circ \quad ; \quad U_{\text{LL}} = \sqrt{3}U_{\text{ph}}$$

Line-to-line voltage is the phasor difference of two phase-to-neutral voltages. It is $\sqrt{3}$ times larger and shifted by 30°.

Star and delta

Connections



In star connection each load sees phase voltage and a neutral point exists. In delta connection each load sees line-to-line voltage; there is no neutral point and circulating currents can occur if the three impedances are not balanced.

$$U_Y = \frac{U_{\text{LL}}}{\sqrt{3}} \quad ; \quad I_Y = I_L \quad ; \quad U_\Delta = U_{\text{LL}} \quad ; \quad I_\Delta = \frac{I_L}{\sqrt{3}}$$

$$\underline{Z}_\Delta = 3\underline{Z}_Y \quad ; \quad \underline{Z}_Y = \frac{\underline{Z}_\Delta}{3}$$

The impedance conversion is valid only for balanced loads or symmetrical network equivalents.

Three-phase power

For balanced operation the total power is three times the phase power. The same formula works for star and delta if line-to-line voltage and line current are used.

$$\underline{S}_{3\text{ph}} = 3\underline{U}_{\text{ph}}\underline{I}^*_{\text{ph}} = \sqrt{3}U_{\text{LL}}I_L\angle\varphi = P + jQ$$

$$S = \sqrt{3}U_{\text{LL}}I_L \quad ; \quad P = \sqrt{3}U_{\text{LL}}I_L\cos\varphi \quad ; \quad Q = \sqrt{3}U_{\text{LL}}I_L\sin\varphi$$

Unbalanced star loads

If the three load impedances differ, the load star point K can shift relative to the supply neutral point N. This changes the phase voltages across the loads and can overvoltage one phase while undervolting another.

$$\underline{Y}_i = \frac{1}{\underline{Z}_i}$$

Without neutral conductor

Without a neutral conductor, the star point is floating. Its displacement is set by the three load admittances.

$$\underline{U}_{KN} = \frac{\underline{Y}_1\underline{U}_{1N} + \underline{Y}_2\underline{U}_{2N} + \underline{Y}_3\underline{U}_{3N}}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3}$$

With neutral impedance

With a neutral conductor, neutral current can flow and the displacement is reduced. An ideal neutral fixes K to N.

$$\underline{U}_{KN} = \frac{\underline{Y}_1\underline{U}_{1N} + \underline{Y}_2\underline{U}_{2N} + \underline{Y}_3\underline{U}_{3N}}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3 + \underline{Y}_N} \quad ; \quad \underline{Y}_N = \frac{1}{\underline{Z}_N}$$

$$\underline{U}_{iK} = \underline{U}_{iN} - \underline{U}_{KN} \quad ; \quad \underline{I}_i = \underline{Y}_i\underline{U}_{iK} \quad ; \quad \underline{I}_N = \underline{I}_1 + \underline{I}_2 + \underline{I}_3$$

Ideal neutral: $\underline{Z}_N = 0 \Longrightarrow \underline{U}_{KN} = 0$.

SW 3: Power Lines & Transformers

Stiff grid

Grid whose voltage magnitude U and frequency f remain approximately constant for the connected equipment. It is modelled as an ideal voltage source behind a very small internal impedance.

$$\Delta U \approx 0 \quad ; \quad \Delta f \approx 0$$

A stiff grid has high short-circuit power: a load or fault current causes only a small voltage change. A weak grid has larger voltage deviations and stronger interaction between connected devices.

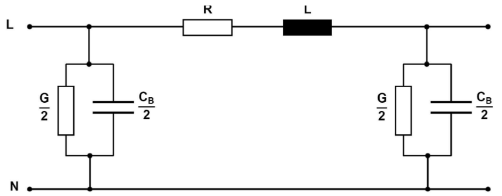
Equivalent line model

Distributed parameters

Power lines are not ideal wires. Resistance causes losses, inductance causes reactive voltage drop, capacitance produces charging current, and conductance represents insulation/leakage losses. These quantities are distributed along the line and are usually given per kilometre.

$$R = R'l \quad ; \quad L = L'l \quad ; \quad G = G'l \quad ; \quad C = C'l$$

$$\underline{Z} = (R' + j\omega L')l \quad ; \quad \underline{Y} = (G' + j\omega C')l$$



Short, medium and long lines

Short lines can often neglect shunt capacitance. Medium lines use a lumped Π model because charging current becomes relevant. Long lines require distributed modelling or cascaded Π sections because voltage and current vary continuously along the line.

Length	Model	Elements
< 100 km	short	series $\underline{Z}$
100–300 km	medium	lumped Π
> 300 km	long	distributed / cascaded Π

Line parameters

Inductance increases with conductor spacing and decreases with larger effective conductor radius. Capacitance behaves oppositely: closer conductors or larger radius increase capacitance. Bundled conductors reduce field strength and improve corona and reactance behaviour.

$$L' = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{r'}\right) \quad ; \quad C' = \frac{2\pi\varepsilon\varepsilon_0}{\ln(D/r')} \quad ; \quad r' \approx 0.78r$$

$$R'_{\text{DC}} = \frac{1}{\kappa A}$$

where: D = conductor spacing; r = radius; κ = conductivity. G' is often neglected.

Skin effect

In AC operation the alternating magnetic field induces eddy currents inside the conductor. They oppose current in the centre and concentrate current near the surface. The usable cross-section becomes smaller, so the effective resistance is higher than the DC resistance.

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}} = \sqrt{\frac{2}{\omega\mu\kappa}} \quad ; \quad R_{\text{AC}} > R_{\text{DC}}$$

where: δ = penetration depth; at 50 Hz $\delta \approx 9.38$ mm. Higher frequency, higher permeability or lower conductivity reduce penetration depth. Large conductors therefore use stranded/bundled designs to limit AC resistance and corona effects.

Surge impedance loading

The surge impedance is the characteristic ratio of voltage to current of a lossless line. At surge impedance loading, the line's inductive reactive power consumption is compensated by its capacitive charging power. The line then neither produces nor absorbs net reactive power.

$$Z_W = \sqrt{\frac{L'}{C'}}$$

For $R' = G' = 0$ and $Z_2 = Z_W$:

$$Q_1 = Q_2 = 0 \quad ; \quad P_1 = P_2 = \frac{|U_1|^2}{Z_W} = \frac{|U_2|^2}{Z_W}$$

Typical overhead line: $Z_W \approx 200\text{–}400\,\Omega$. Below SIL the line tends to generate reactive power; above SIL it tends to consume reactive power and voltage support becomes more critical.

Voltage drop along a line

For overhead lines, reactance usually dominates resistance. Active power mainly affects the load angle, while reactive power mainly affects voltage magnitude. Inductive loads therefore lower the receiving-end voltage; capacitive compensation can support it.

$$\underline{U}_1 = \underline{U}_2 + jX\underline{I}_2 \quad ; \quad \underline{S}_2 = P_2 + jQ_2 = \underline{U}_2\underline{I}_2^*$$

$$U_1 \approx U_2 + \frac{XQ_2}{U_2} \quad ; \quad \delta \approx \frac{XP_2}{U_2^2}$$

Transformers

Function in the grid

Transformers couple grid levels without changing frequency. They allow high voltage for low-loss transmission and lower voltage for distribution and consumer safety. Ideally, power is conserved and only voltage/current levels change.

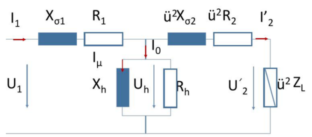
Ideal and referred quantities

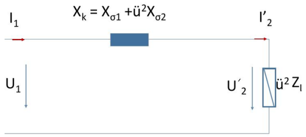
$$\ddot{u} = \frac{N_1}{N_2} = \frac{\underline{U}_1}{\underline{U}_2} = \frac{\underline{I}_2}{\underline{I}_1} \quad ; \quad \underline{S}_1 = \underline{S}_2$$

$$\underline{U}_2' = \ddot{u}\,\underline{U}_2 \quad ; \quad \underline{I}_2' = \frac{\underline{I}_2}{\ddot{u}} \quad ; \quad \underline{Z}_2' = \ddot{u}^2\underline{Z}_2$$

Referred quantities move the secondary side to the primary side so the whole transformer can be calculated as one equivalent circuit. Impedances transform with the square of the turns ratio.

Equivalent circuit





The series resistance represents copper losses. Leakage reactance represents flux that links only one winding and causes voltage drop under load. The shunt branch represents magnetising current and iron losses; it is often neglected in short-circuit or load-flow approximations.

$$R_2' = \ddot{u}^2 R_2 \quad ; \quad X_{\sigma 2}' = \ddot{u}^2 X_{\sigma 2}$$

$$R_k = R_1 + R_2' \quad ; \quad X_k = X_{\sigma 1} + X_{\sigma 2}' \quad ; \quad \underline{Z}_k = R_k + jX_k$$

Relative short-circuit voltage

The short-circuit voltage describes the transformer impedance in a compact rating-based form. A higher u_k limits fault current better but causes a larger voltage drop in normal operation.

$$u_k = \frac{U_{1k}}{U_{1N}} \quad ; \quad |\underline{Z}_k| = u_k\frac{U_N^2}{S_N}$$

u_k: primary-voltage fraction producing nominal current with the secondary short-circuited; typical range 4–18%.

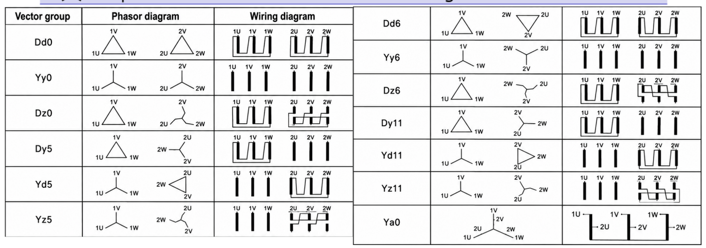
Three-phase transformers

Connections and vector groups

Star, delta and zig-zag connections determine neutral availability, insulation stress, zero-sequence behaviour and phase shift. Delta windings can provide a path for triplen harmonics and zero-sequence currents; star windings can expose a neutral point.

Circuit	Symbol	Character HV	Character LV	Application
Star	⅄	Y	y	High rated voltage
Delta	△	D	d	High rated current
Zick-Zack		Z	z	1-phase loads

HV	LV	Vector group	Application
⅄	⅄	Yy0	Between 2 HV grids
⅄	△	Yd5	Between generator and HV grid
⅄		Yz5	Feed-in to LV grid



The vector-group clock number gives the LV line-voltage displacement relative to the HV side in multiples of $30°$.

$$\underline{u} = \frac{\underline{U}_{OS}}{\underline{U}_{US}} = \frac{\underline{U}_{1U1V}}{\underline{U}_{2U2V}} = \frac{\underline{U}_{1U1N}}{\underline{U}_{2U2N}}$$

SW 4: Synchronous Generators

Three-phase generation

Basic principle

A synchronous generator converts mechanical torque into three-phase electrical power. The rotor produces a magnetic field by DC excitation or permanent magnets. The stator has three windings shifted by $120°$; as the rotor field turns, it induces three sinusoidal voltages.

$$U(t) = B \cdot A \cdot \omega \cdot \sin(\omega t) \quad ; \quad n = \frac{60f}{p} \quad ; \quad \omega_{\text{mech}} = \frac{\omega}{p} = \frac{2\pi f}{p} = 2\pi\frac{n}{60}$$

where: n = speed [rpm]; p = number of pole pairs. Non-salient machines are used at high speed; salient-pole machines use many poles for low-speed drives.

Non-salient vs. salient-pole generators

	Non-salient pole	Salient pole
Rotor	cylindrical, nearly constant air gap	projecting poles, variable air gap
Speed	high: typically 1500/3000 rpm	low: typically ≤ 1000 rpm
Pole pairs	few: usually 1–2	many
Drive	steam/gas turbine	hydro turbine
Model	$X_d = X_q$	$X_d \neq X_q$

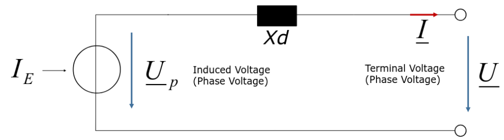
Non-salient rotors are mechanically robust at high speed and are typical for steam or gas turbines. Salient-pole rotors are suited to low-speed hydro turbines because many pole pairs allow synchronous operation at low rpm.

Grid inertia

Rotating synchronous masses release kinetic energy after a power imbalance, slowing the initial frequency change. This gives control systems time to react. Converter-based generation is decoupled from mechanical inertia unless a synthetic/virtual inertia function is implemented.

Equivalent circuit and torque angle

For stationary balanced operation of a non-salient generator, neglecting stator resistance:



$$X_d = X_m + X_\sigma \quad ; \quad \underline{U}_p = \underline{U} + jX_d\underline{I} \quad ; \quad \underline{I} = \frac{\underline{U}_p - \underline{U}}{jX_d}$$

where: $\underline{U}_p$ = induced phase voltage; $\underline{U}$ = terminal phase voltage; X_d = synchronous reactance.

$$\vartheta = \varphi_{U_p} - \varphi_U$$

$\vartheta > 0$: generator operation; $\vartheta < 0$: motor operation; $\vartheta = 0$: no active power. The torque angle is the electrical angle between internal generated voltage and terminal voltage. Increasing mechanical input increases ϑ and active power until the stability limit is reached.

Active and reactive power

For a grid-connected generator, the grid fixes terminal voltage and frequency. Mechanical torque mainly controls active power. Excitation changes the internal voltage U_p and mainly controls reactive power.

With $\underline{U} = U\angle 0°$ and $\underline{U}_p = U_p\angle\vartheta$:

$$P = \frac{UU_p}{X_d}\sin\vartheta \quad ; \quad Q = \frac{UU_p}{X_d}\cos\vartheta - \frac{U^2}{X_d}$$

$$P_{\text{3ph}} = 3P \quad ; \quad Q_{\text{3ph}} = 3Q$$

$$P_{\text{max,3ph}} = \frac{3UU_p}{X_d} \quad \text{at} \quad \vartheta = 90°$$

Operation near $90°$ is not acceptable in practice because synchronism would be lost after a small disturbance. Real generators therefore operate with a stability margin.

Operating modes and excitation

- **Island operation**: generator sets voltage and frequency; excitation controls voltage and turbine/governor controls frequency
- **Grid connected**: grid fixes U and f; mechanical input mainly controls P, excitation mainly controls Q
- **Over-excited**: $U_p > U$, supplies Q (capacitive behaviour)
- **Under-excited**: $U_p < U$, absorbs Q (inductive behaviour)

At $\vartheta = 0$:

$$P_{\text{3ph}} = 0 \quad ; \quad Q_{\text{3ph}} = \frac{3}{X_d}\left(U_p U - U^2\right)$$

Over-excitation raises the internal voltage and supports the grid voltage by supplying reactive power. Under-excitation absorbs reactive power and can lower the voltage.

Grid synchronization

Connection to the grid requires:

1. equal frequency
2. equal phase position
3. equal voltage magnitude
4. equal phase sequence L1–L2–L3

If these criteria are not fulfilled, closing the breaker causes large transient currents and mechanical torque shocks. Synchronisation is therefore checked before paralleling a generator to the grid.

Per-unit system

Referenced quantities

The per-unit system expresses quantities relative to selected base values. It removes many transformer ratio conversions and makes impedances comparable across voltage levels.

$$u = \frac{U}{U_N} \quad ; \quad i = \frac{I}{I_N} \quad ; \quad s = \frac{S}{S_N} \quad ; \quad z = \frac{Z}{Z_N}$$

Per-unit values are dimensionless. Choose U_N and S_N; the remaining base values follow.

Three-phase base values

Using line-to-line U_N and total three-phase S_N:

$$I_N = \frac{S_N}{\sqrt{3}U_N}$$

$$Z_{N,Y} = \frac{U_N^2}{S_N} \quad ; \quad Z_{N,\Delta} = 3\frac{U_N^2}{S_N}$$

$$Z = zZ_N$$

For the usual star-based network reference:

$$Z = z\frac{U_N^2}{S_N} \quad ; \quad z = Z\frac{S_N}{U_N^2}$$

SW 5: Grid Structure / Load Flow

Grid structures

Transmission and distribution grids

The transmission grid is strongly meshed: each node represents a substation or connection point, and several paths exist between major nodes. This improves reliability and allows power to be rerouted after outages, but requires automation, protection coordination and load-flow monitoring.

Medium-voltage distribution grids transfer energy from the high-voltage grid to local low-voltage grids. In populated areas they are often built as cables. Even if the physical structure is a ring, normal operation is frequently radial with one open switch to simplify protection and fault localisation.

Distribution topologies

Topology	Advantages	Disadvantages
radial	cheap, simple protection	low supply reliability
ring	feeder switching possible, higher reliability	more switches and planning
meshed	highest reliability, many supply paths	expensive, complex protection

Low-voltage networks in Europe usually provide 400 V for three-phase loads and 230 V for single-phase loads. They are built as four-wire systems so single-phase consumers can be connected between phase and neutral.

Representation of power grids

Nodes and branches

A **node** is a branch point or connection point of lines, transformers, loads or generators. A **branch** connects two nodes and represents a line, transformer, switch or equivalent impedance.

Quantity	Meaning
P_A, Q_A	active/reactive power injected or consumed at node A
U_A, δ_A	voltage magnitude and phase angle at node A
P_{AB}, Q_{AB}	active/reactive power flow from A to B
$P_{\text{loss}}, Q_{\text{loss}}$	branch losses

The complete network state is known when all node voltages and branch flows are known. Load-flow analysis checks line loading, bottlenecks, voltage violations and losses in a given operating point.

Branch model

Branches can be represented as a four-pole or by a Π equivalent. For lower voltage and short distribution lines, shunt capacitance is often neglected and the line is modelled only by a series impedance.

$$\underline{Z} = R + jX \quad ; \quad \underline{I}_{AB} = \frac{\underline{U}_A - \underline{U}_B}{\underline{Z}}$$

Cables and long/high-voltage lines need shunt admittance because charging current influences voltage and reactive power.

Two-node load flow

For a simple feeder with known sending-end voltage $\underline{U}_A$ and known load power $\underline{S}_B = P_B + jQ_B$, the receiving-end voltage $\underline{U}_B$ is usually unknown. This is why the problem is nonlinear: current depends on voltage, but voltage depends on current.

$$\underline{I}_B = \frac{\underline{S}_B^*}{\underline{U}_B^*} \quad ; \quad \underline{U}_B = \underline{U}_A - \underline{Z}\underline{I}_B$$

Inductive loads consume positive Q in load convention and increase voltage drop; **capacitive** behaviour can reduce or reverse the reactive voltage drop.

Node types

Load-flow problems need two specified quantities per node. The remaining two are solved by the load-flow equations.

Node type	Given	Unknown
slack/reference	U, δ	P, Q
PU / PV node	P, U	Q, δ
PQ load node	P, Q	U, δ

Slack node: fixes the voltage-angle reference and balances the power mismatch caused by losses.
PU node: typical generator with voltage regulation; if reactive limits are reached, it may behave as a PQ node.
PQ node: typical load without voltage regulation.

Admittance matrix

Construction rules

The bus admittance matrix links node currents and node voltages. It is the key network model for numerical load-flow algorithms.

$$\underline{\mathbf{I}} = \underline{\mathbf{Y}}_{\text{bus}}\underline{\mathbf{U}}$$

For passive reciprocal networks, $\underline{\mathbf{Y}}_{\text{bus}}$ is symmetric.

- **Diagonal**: $\underline{Y}_{ii}$ is the sum of all admittances connected to node i, including shunts.
- **Off-diagonal**: $\underline{Y}_{ij}$ is the negative sum of admittances directly connecting nodes i and j.
- **No connection**: $\underline{Y}_{ij} = 0$.

$$\underline{y}_{ij} = \frac{1}{\underline{z}_{ij}} \quad ; \quad \underline{Y}_{ii} = \sum \underline{y}_{ij} + \underline{y}_{\text{sh},i} \quad ; \quad \underline{Y}_{ij} = -\underline{y}_{ij}$$

Node current and power

For node k:

$$\underline{I}_k = \sum_{i=1}^{n} \underline{Y}_{ki}\underline{U}_i \quad ; \quad \underline{S}_k = \underline{U}_k\underline{I}_k^*$$

With $\underline{U}_i = U_i\angle\delta_i$ and $\underline{Y}_{ki} = Y_{ki}\angle\Theta_{ki}$:

$$P_k = \sum_{i=1}^{n} Y_{ki}U_kU_i \cos(\Theta_{ki} + \delta_i - \delta_k)$$

$$Q_k = -\sum_{i=1}^{n} Y_{ki}U_kU_i \sin(\Theta_{ki} + \delta_i - \delta_k)$$

These nonlinear equations are written for all non-slack node quantities that are specified by the node type.

Voltage-dependent PQ nodes

Real loads are not always constant-power loads. Their voltage dependence changes the load-flow result and the voltage-stability behaviour.

Load model	Voltage behaviour
constant power	P, Q fixed; current rises when voltage falls
constant current	P, Q approximately proportional to U
constant impedance	P, Q approximately proportional to U^2

SW 6: Grid Operation

Design criteria and grid services

Secure operation

Grid operation keeps the system inside technical limits at every time. Security means that the grid can tolerate expected disturbances without cascading outages.

- frequency within limits
- node voltages within admissible bands
- no overload of lines, transformers or generators
- short-circuit proof equipment and protection coordination
- static and dynamic stability, considering the $N-1$ criterion

Ancillary services

Ancillary services are mechanisms used to maintain secure operation or restore it after a disturbance. TSOs normally procure these services from market participants because they do not own generation or controllable loads.

Service	Purpose
frequency control	balance active power and keep $f \approx 50$ Hz
voltage support	provide/absorb reactive power to keep voltages in range
loss compensation	cover active power losses in the grid
black start	restart grid islands without external supply
system coordination	schedules, congestion management, measurements

Frequency control

Active power balance

Frequency is the global indicator of active-power balance. If generation exceeds load, synchronous machines accelerate and frequency rises. If load exceeds generation, rotating masses decelerate and frequency falls.

$$\Delta P = P_{\text{gen}} - P_{\text{load}}$$

During normal operation, frequency is almost the same throughout the synchronous area because active power is transported comparatively easily over long distances. A stable frequency is required for machines, converters, protection systems and clocks based on grid frequency.

Inertia and frequency-dependent load

Immediately after a disturbance, kinetic energy stored in synchronously connected rotating masses counteracts the frequency change. This inertial response is inherent and acts before controllers react.

Some loads are frequency-dependent: motors, for example, change their power consumption when speed/frequency changes. This provides a natural damping effect.

$$\Delta P_{\text{demand}} = D\,\Delta f$$

where: D = load-frequency sensitivity, typically about 0–4%/Hz.

Control stages

Stage	Time	Function
inertia	instant	slows initial rate of change of frequency
primary / FCR	≤ 30 s	automatic local stabilisation
secondary / aFRR	~ 5–15 min	restores frequency and area balance
tertiary / mFRR	~ 15 min	manual/operator reserve replacement
time control	slow	corrects accumulated time deviation

Primary control

Primary control is decentralised and activated automatically by local frequency measurement at power plants or controllable units. It stabilises frequency but does not normally restore it exactly to 50 Hz.

$$\Delta P_{\text{gen}} = -R\,\Delta f$$

where: R = droop/regulating characteristic [MW/Hz]. FCR is usually procured symmetrically and must react without artificial delay; typical requirements are partial activation after a few seconds and full activation after about 30 s.

Secondary control

Secondary control is activated centrally by the TSO through automatic generation control. It restores the frequency and the scheduled power exchange between control areas. It acts in the control area where the imbalance occurred.

$$\Delta P_{Ri} = \Delta P_i - \Delta P_{Ki} \quad ; \quad \Delta P_i = -K_{Ni}\Delta f$$

where: ΔP_{Ri} = secondary-control request of area i; ΔP_{Ki} = unscheduled exchange with neighbouring areas; K_{Ni} = network power-frequency characteristic [MW/Hz].

Tertiary and time control

Tertiary control is activated by the operator to free secondary reserves and handle longer-lasting imbalances. Providers can be domestic or foreign.

Time control corrects accumulated time deviation. If the synchronous time deviation becomes too large, the frequency setpoint of automatic generation control is slightly adjusted.

Reserve requirements

Primary and secondary control products are prequalified by Swissgrid. Important exam distinction: FCR reacts directly to frequency deviation; aFRR follows a TSO setpoint and restores the control area balance.

Reserve	Activation	Typical procurement
FCR / primary	proportional to Δf	daily blocks, symmetric bids
aFRR / secondary	AGC setpoint from TSO	weekly, positive/negative bands
tertiary	operator request	temporary contracts / balancing market

Load shedding

If frequency continues to fall and automatic reserves cannot stabilise the system, loads are disconnected automatically. This is a last-resort protection measure: losing some load is preferable to losing synchronism of the whole system.

Load shedding can preserve regional islands with sufficient local generation. It is organised by ENTSO-E members and must be autonomous, because operator reaction would be too slow during severe frequency collapse.

Voltage and reactive power control

Voltage behaviour

Ideally, the voltage profile is flat: node voltages remain close to nominal and the grid is mainly used for active-power transport. In reality, voltages depend on loading, reactive power flows and transformer tap positions.

Reactive power has a local effect because transmission lines and transformers are mainly inductive ($X \gg R$). Therefore Q cannot be transported over long distances efficiently.

- supplying capacitive reactive power increases nearby voltage
- absorbing inductive reactive power lowers nearby voltage
- reactive current increases loading and voltage drops

Reactive power sources and sinks

Supply of Q	Absorption of Q
over-excited synchronous machines	under-excited synchronous machines
capacitor banks	induction motors
line/cable capacitance	line/cable inductance
synchronous condensers / SVC	transformer inductance / reactors

Voltage-control options

- generator voltage control by excitation
- synchronous condensers: synchronous machines without mechanical load
- switched capacitor banks or reactors for load-dependent compensation
- SVC/FACTS for fast continuous reactive-power control
- transformer on-load tap changer (OLTC) to change voltage ratio under load

Swissgrid distinguishes semi-active voltage regulation, often mandatory for lower-grid DSOs, and active voltage regulation with larger compensation and prequalification requirements for power plants.

Virtual power plants and flexibility

Ancillary services were traditionally provided by large power plants, large industrial consumers or emergency generators. A virtual power plant aggregates many smaller flexible devices into one controllable pool.

- examples: heat pumps, batteries, micro-generators, flexible loads, V2G
- strengths: high aggregated power, short response time, low energy need
- use cases: balancing, congestion management, voltage support, arbitrage, self-consumption optimisation, peak shaving

Blackout Italy 2003

Cascading mechanism

On 28 September 2003 Italy suffered a nationwide blackout. The initial problem was not one single missing reserve, but a cascade of line trips and loss of synchronism with the interconnected European system.

1. **Line overload**: high power flow heated conductors.
2. **Conductor sag**: higher temperature increased sag.
3. **Flashover**: reduced clearance caused contact/flashover to trees.
4. **Line trip**: remaining lines carried more power and overloaded.
5. **Cascade**: further trips separated Italy from the UCTE system.

Frequency collapse

At the time, Italy imported large power from Austria, Switzerland and France. After separation, local generation was insufficient; a deficit of about 6.6 GW occurred. Primary control, disconnection of pumping units and load shedding slowed the frequency drop, but generation units also tripped.

When frequency fell below about 47.5 Hz, remaining generating units disconnected to protect themselves. The result was an unavoidable blackout. Key lesson: thermal limits, vegetation clearance, protection settings, reserves and inter-area coordination all interact in system security.

SW 7: Short Circuits / Protection Systems

Substations

GIS and AIS

Substations connect, transform, switch and protect grid sections. Their layout depends on voltage level, available space, environmental conditions and reliability requirements.

	GIS	AIS
Meaning	gas-insulated substation	air-insulated substation
Pros	compact, expandable, good for urban areas	cheaper, simple setup, air insulation
Cons	expensive, costly maintenance	large space, exposed to weather/pollution

Main substation elements

Element	Function
busbar	common node connecting feeders, transformers and couplers
disconnector	visible isolation; not for interrupting fault current
circuit breaker	interrupts load and short-circuit currents
instrument transformer	adapts current/voltage for measurement and protection
overvoltage protection	limits lightning and switching overvoltages
power transformer	couples voltage levels

Protection fundamentals

Purpose and selectivity

Protection systems detect faults and isolate only the faulty element as fast as possible. The goal is to minimise danger, equipment damage and supply interruption while keeping the healthy grid in operation.

Selectivity: the protection device closest to the fault trips first. Upstream devices act as backup with longer delay or higher threshold. In radial grids this is usually achieved by current/time grading; in meshed grids, directional, differential or distance criteria are needed.

Protection families

Type	Typical use
fuses	passive, simple LV/MV short-circuit protection
overcurrent	radial feeders, graded time delays
differential	generators, transformers, busbars, short cables
distance	HV lines; impedance-based fault location
I_S limiter	limits surge short-circuit current

Fuses

HH and LH fuses

Fuses convert fault current into heat and melt when the generated heat exceeds the fuse characteristic. They are simple and fast, but single-use and less flexible than relay-based protection.

	HH fuse	LH fuse
Level	MV, about 1–36 kV	LV, e.g. 400 V networks
Main role	short-circuit protection	overload and short-circuit protection
Behaviour	trips only at high multiples of rated current	must interrupt from melting current to rated breaking current

Fuse selectivity

In radial networks, rated fuse currents should differ by at least about a factor of 1.6 to obtain selectivity. In meshed LV networks, identical LH fuses may be used, but the short-circuit current at the selective device must be high enough compared with the neighbouring path; proper neutral treatment is important.

Relay-based protection

Measure–compare–switch

Active protection systems work in three steps:

1. **Measure**: instrument transformers acquire currents/voltages.
2. **Compare**: relay evaluates values against settings and logic.
3. **Switch**: circuit breaker isolates the faulty component.

The circuit breaker must be rated for the maximum expected fault current, while relay settings must still detect the minimum fault current.

Differential protection

Differential protection compares currents entering and leaving a protected zone. For a healthy zone, the current sum is approximately zero; for an internal fault, the difference exceeds the threshold.

$$|\underline{I}_1 - \underline{I}_2| > I_{\text{threshold}}$$

It is selective by design because it trips only for faults inside its protected zone. It requires communication/signal lines and is used for transformers, generators, busbars and short lines or cables.

Overcurrent protection

DMT protection trips if the measured current exceeds a set value for a set delay time. Several relays in series can be coordinated by increasing the trip delay toward the source.

Pros: simple, cheap, robust. **Cons**: selectivity is harder in meshed grids and the threshold must distinguish high load current from minimum short-circuit current.

Distance protection

Distance protection estimates the fault distance indirectly by measuring the impedance from the relay location to the fault.

$$\underline{Z}_L = (R'_L + jX'_L)\,l \quad ; \quad \underline{Z}_{\text{meas}} = \frac{\underline{U}}{\underline{I}}$$

Small measured impedance means a nearby fault and can trip immediately; larger impedance zones trip with delay. Directional criteria ensure that the relay trips only for faults in the protected direction.

Fault analysis

Why short-circuit currents are calculated

Short-circuit analysis is required for breaker rating, protection settings and network expansion planning. It must be repeated when topology changes, because new generators, lines or transformers can significantly increase fault levels.

Important setting conflict: protection must not trip during heavy load, but must still trip for the minimum short-circuit current at the end of the protected zone.

Fault types and consequences

Short circuits may be solid or include fault impedance/arc resistance. Most transmission-line faults are single-phase-to-earth faults, but symmetrical three-phase faults often produce the highest current and are simpler to calculate.

- disturbance of nearby communication lines
- stability problems due to voltage dips and acceleration of machines
- mechanical forces on busbars, windings and conductors
- thermal stress and possible equipment destruction
- danger to personnel from touch/step voltages and arcs

Short-circuit forces rise with current and can mechanically damage conductors, windings, shafts and foundations.

Symmetrical short-circuit current

RL fault transient

For a simplified line or transformer leakage path, the short-circuit current is the sum of a symmetrical AC component and a decaying DC offset.

$$v(t) = \sqrt{2}\,V_S \sin(\omega t + \alpha) \quad ; \quad i(t) = i_s(t) + i_t(t)$$

$$i_s(t) = \frac{\sqrt{2}\,V_S}{Z} \sin(\omega t + \alpha - \Theta)$$

$$i_t(t) = -i_s(0)e^{-Rt/L} = \frac{\sqrt{2}\,V_S}{Z} \sin(\Theta - \alpha)e^{-Rt/L}$$

$$Z = \sqrt{R^2 + \omega^2 L^2} \quad ; \quad \Theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

where: V_S = RMS source voltage; α = switching/fault inception angle. The DC offset depends on the instant of fault inception and decays with time constant L/R.

Maximum momentary current

The worst asymmetry occurs when the DC offset adds to the AC peak. A useful upper bound is:

$$i_{\text{mm}} \leq 2\frac{\sqrt{2}\,V_S}{Z}$$

Circuit breakers must interrupt the AC short-circuit current and withstand the mechanical stress from the peak/momentary current.

Unsymmetrical faults

Symmetrical components

Most real faults are unsymmetrical. They are calculated with symmetrical components: any unbalanced three-phase sinusoidal system can be decomposed into three balanced systems.

Sequence	Meaning
positive	normal phase order; transfers useful three-phase power
negative	opposite phase order; appears during unbalance
zero	three phases in phase; needs earth/neutral return path

This course mainly uses symmetrical three-phase fault calculations, but the method is important because earth faults and phase-to-phase faults require sequence networks.

Neutral point connection

Purpose

Neutral-point treatment is usually implemented at transformer neutrals. It has little influence during balanced normal operation, but strongly affects earth fault current, healthy-phase overvoltages and protection behaviour.

- limit touch and step voltages
- limit voltages on healthy conductors during earth faults
- limit fault current and enable arc self-extinction
- allow selective earth-fault detection and maintain supply if possible

Comparison

Treatment	Characteristics	Application
isolated neutral	low earth-fault current, high healthy-phase voltages, arcs may self-extinguish	CH medium voltage, small networks
earth-fault compensation	Petersen coil compensates capacitive earth-fault current, adjustable after reconfiguration	DE medium voltage
low-impedance grounding	high fault current, little healthy-phase overvoltage, needs fast protection	≥ 110 kV and LV grids

Tradeoff: low fault current reduces damage and can keep supply running, but makes fault detection harder and increases healthy-phase voltage. Low-impedance grounding gives clear protection signals but requires fast disconnection.

SW 8: Thermal Power Plants

Energy outlook and Swiss context

Demand and production

Swiss electricity demand is expected to increase strongly by 2050 because of electrification, especially EVs, heat pumps and sector coupling such as power-to-gas and district heating.

Swiss production is already low-CO_2: more than 90% comes from hydro and nuclear power. Production is highly seasonal. Run-of-river plants produce most in summer and can create export surplus, while winter supply relies more on reservoir hydro and electricity imports.

Gas turbines and CCPP

Gas turbines

In a gas turbine, ambient air is compressed, mixed with fuel and burned at high temperature. The hot gas expands through the turbine and drives both the compressor and generator.

$$\eta_{\text{GT}} \approx 40\% \quad ; \quad t_{\text{start}} \approx 5\text{–}10 \text{ min}$$

Gas turbines are useful as flexible reserve and peak units because they start quickly, but their standalone efficiency is limited by hot exhaust losses.

Combined-cycle power plants

A combined-cycle plant uses the gas-turbine exhaust in a heat-recovery steam generator. The generated steam drives a second steam turbine.

$$\eta_{\text{CCPP}} \approx 60\%$$

- **Brayton part**: gas turbine produces fast electrical output.
- **Rankine part**: waste heat boiler and steam turbine recover exhaust heat.
- **Swiss reserve role**: winter reserve plants may use CO_2-neutral fuels such as HVO, synthetic methane or methanol.

Nuclear power plants

Role and operation

Nuclear plants supply base-load power. They generate roughly one third of Swiss electricity, with almost CO_2-free operation. Under the Energy Strategy 2050, no new nuclear plants may be built, but existing plants can continue as long as they are considered safe.

Reactor types

Type	Principle
PWR	two circuits; primary water is kept at about 160 bar so it does not boil; heat is transferred to a secondary steam circuit
BWR	one circuit; water boils directly in the reactor vessel and steam goes to the turbine

PWR examples: Beznau, Goesgen. BWR example: Leibstadt. BWRs operate at lower pressure, but radioactive steam reaches the turbine hall, so shielding requirements are stricter.

Waste incineration

Waste incineration plants are classic base-load units with high availability. They burn waste at about 1000°C and generate superheated steam. They are not suited to fast grid stabilisation because the furnace and boiler have large thermal inertia.

About half of the waste is biogenic, so that part is often counted as renewable. Waste heat is valuable for district heating; combined heat and power operation increases overall efficiency.

Geothermal and fusion energy

Geothermal

Type	Characteristics
hydrothermal	uses natural hot-water aquifers; ORC efficiency only about 12%, but low fuel cost and base-load output
petrothermal / EGS	drills 3000–6000 m into hot dry rock; high-pressure injection creates an artificial heat exchanger

Hydrothermal heat is especially attractive for district heating. Petrothermal systems have large potential but are technically complex and can induce earthquakes.

Fusion

Tokamak reactors aim to fuse deuterium and tritium into helium. They promise very high safety, no meltdown mechanism, no greenhouse gases and no long-lived radioactive waste. Magnetic confinement uses toroidal, poloidal and central solenoid fields to shape, stabilise and drive the plasma.

SW 9: Hydropower Plants

Load profiles and the duck curve

Base load and peak load

Run-of-river, nuclear and conventional thermal plants provide constant base load but react slowly to rapid demand changes. Storage hydropower is flexible and therefore well suited to cover dynamic peak loads.

Duck curve

With high PV capacity, residual grid load drops strongly around midday and then rises quickly in the evening when solar production falls and demand peaks.

$$P_{\text{res}} = P_{\text{load}} - P_{\text{PV}} - P_{\text{wind}}$$

The steep evening ramp stresses balancing resources and requires fast dispatchable generation, storage or demand response.

Hydropower classification

Head

The head is the elevation difference between headwater and tailwater. It determines whether energy is mainly harvested from pressure/head or from very large flow volumes.

Plant	Characteristics
low-head	up to about 25 m; large flows; run-of-river, weir and powerhouse in or near river channel
high-head	about 50 m to over 1000 m; smaller flow at high pressure; dam, intake, penstock and surge tank

Surge tanks reduce pressure waves when valves close quickly.

Turbine technologies

Turbines are selected mainly from head and volumetric flow.

Turbine	Best range	Principle
Pelton	very high head > 300 m, low flow	water jets hit cup-shaped buckets; impulse turbine
Francis	medium head ≈ 25–700 m	guide vanes feed a reaction runner; all-round turbine
Kaplan	low head < 50 m, high flow	propeller-like runner with adjustable blades

$$P_{\text{hydr}} = \rho g Q h \quad ; \quad P_{\text{el}} = \rho g Q h \, \eta_T \eta_G$$

Sustainability and ecology

Hydropower is renewable, reliable and CO_2-free, but dams interrupt river ecosystems.

- **Fish migration**: dams block spawning routes; mitigation uses fish ladders, lifts and bypass channels. Fine screens protect downstream fish from turbines.
- **Sediment transport**: reservoirs trap bed load, causing silting upstream and sediment starvation downstream. Solutions include by-

pass tunnels, flushing through bottom outlets and dredging.

Tidal and pumped-storage power

Tidal power

Tidal barrage plants trap water at high tide and discharge it through bidirectional turbines at low tide. They need at least about 5–7 m tidal range and yield only about 25–40% of an equivalent river plant.

Tidal-stream turbines use the kinetic energy of tidal currents directly. They are highly predictable and usually have lower ecological impact than barrages.

Pumped storage

Pumped-storage plants perform energy arbitrage: they pump water uphill when electricity is cheap or surplus, and turbine it when prices and demand are high.

$$\eta_{\text{cycle}} = \frac{W_{\text{out}}}{W_{\text{in}}} \approx 80\%$$

Despite losses, they are system-relevant for fast balancing, frequency control and absorbing renewable surplus that would otherwise be curtailed. Pumped energy does not receive green Guarantees of Origin because the input electricity source may be mixed.

SW 10: Smart Grid Trends

Core problem: collision zone

Swiss PV capacity is growing rapidly, with a target of about 39 GWp by 2050. Historically, the Swiss peak grid load is only about 10 GW. The **collision zone** is the range where installed PV capacity exceeds the maximum grid load.

At summer midday, local PV generation can exceed local consumption and heavily load distribution grids in the reverse direction. The main bottleneck is often not national energy balance but local transformers and lines.

Dynamic grid tariffs

Time-of-use and dynamic grid tariffs create price signals to shift flexible consumption away from critical grid periods.

- electricity is cheaper when local solar generation is high or the grid has surplus capacity
- electricity is more expensive during evening peaks or local congestion
- flexible loads: EVs, heat pumps, batteries, appliances and industrial processes

The goal is grid relief, not cheaper electricity production.

Capacity tariffs

Traditional billing mainly charges consumed energy in kWh. Capacity tariffs add a fee based on the maximum drawn power in kW, often measured over a rolling 15-minute window.

$$C_{\text{bill}} \sim E_{\text{kWh}} + P_{\text{max,kW}}$$

This penalises short extreme spikes, e.g. fast EV charging at 11 kW, and rewards slower steady consumption, e.g. charging overnight at 3.7 kW. The effect is peak shaving and reduced bottlenecks in local transformers and lines.

Local energy communities

Local energy communities group consumers and producers so local PV can be used locally. Allocation mechanisms can be fixed, proportional, dynamic or hierarchical.

Model	Meaning
ZEV	joint self-consumption behind one meter on one parcel/building; saves all grid fees internally but needs private wiring
vZEV	virtual ZEV using existing public grid inside a local transformer area; DSO billing enables reduced local grid fee
LEG	local energy community at neighbourhood/municipal scale; uses public grid and legal contracts, no private digging

Allocation keys

- **Fixed**: predefined shares, simple but less adaptive.
- **Proportional**: PV is shared according to simultaneous demand.
- **Dynamic**: allocation optimises current objectives such as peak reduction.
- **Hierarchical**: priority loads or investors are served first, then remaining energy is shared.

ZEV offers the largest grid-fee saving but high private infrastructure effort. vZEV reduces installation cost by using the existing grid locally. LEG has the largest geographic scope and enables peer-to-peer sharing at scale, but members usually still pay regular grid fees.